# Statistical philosophy

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http://www.ici3d.org/mmed/

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- **WARNING: Long Lecture**

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  - *Confirmation:* In Cape Town, it rains more on Sundays than other days
  - *Estimation:* In Cape Town, the *odds* of rain on Sunday are 1.6–2.2 times higher than on other days
  - *Prediction:* I am confident that it will rain at least one Sunday while I am here

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  - Is it likely they *didn't*?

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  - *Range of estimates:* how much do we think the supplement is helping?

# Outline

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- *P values are over-rated*
- *Never use a P value as evidence that an effect is small, or that two quantities are similar.*

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    - Or *control for* other factors

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# What do we hope to learn?

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- How sure are we about that?

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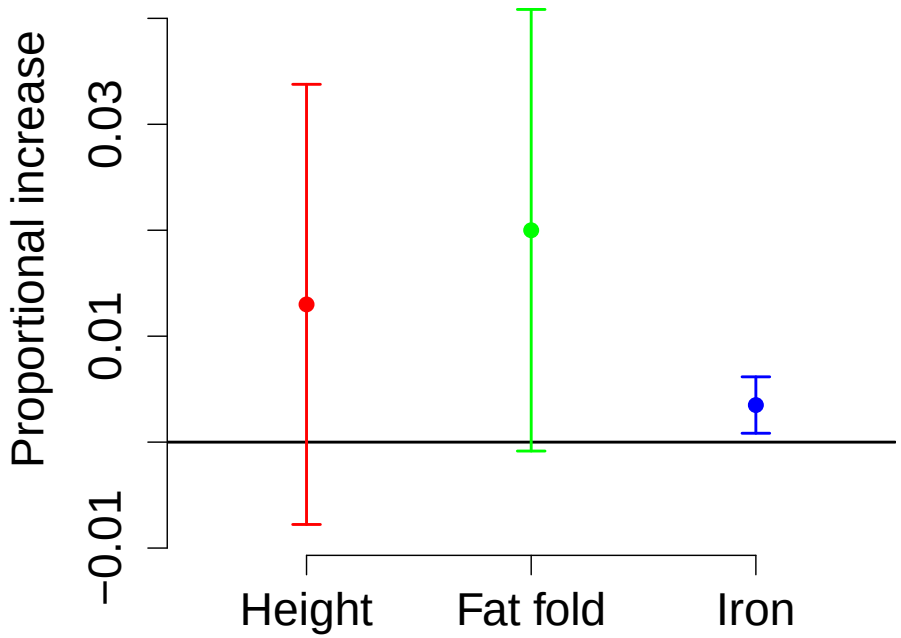
- The difference is unlikely to be due to chance
  - So what! I already know vitamin A has strong effects on metabolism
- If I'm certain that the true answer isn't exactly zero, why do I want the P value anyway?

# Confidence intervals



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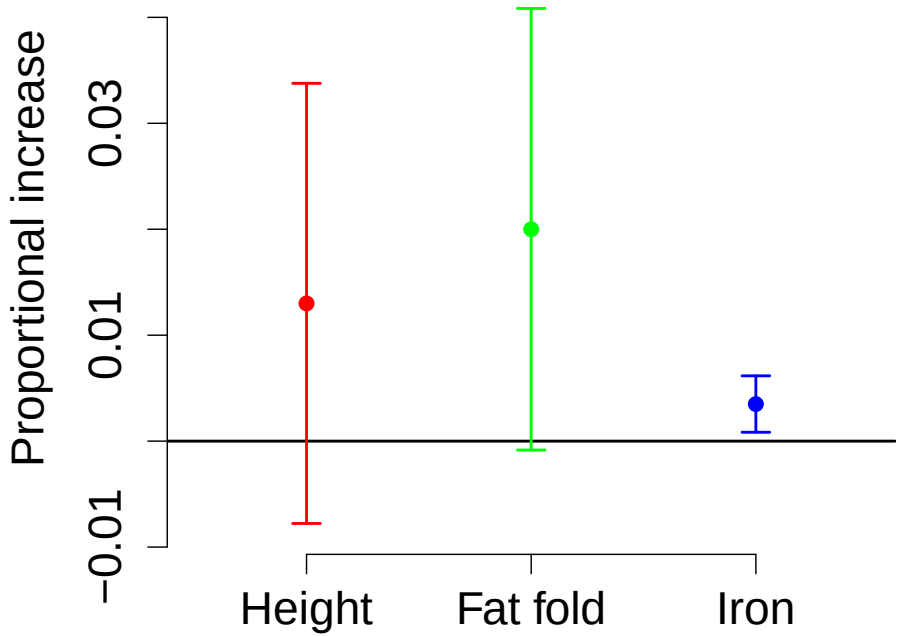


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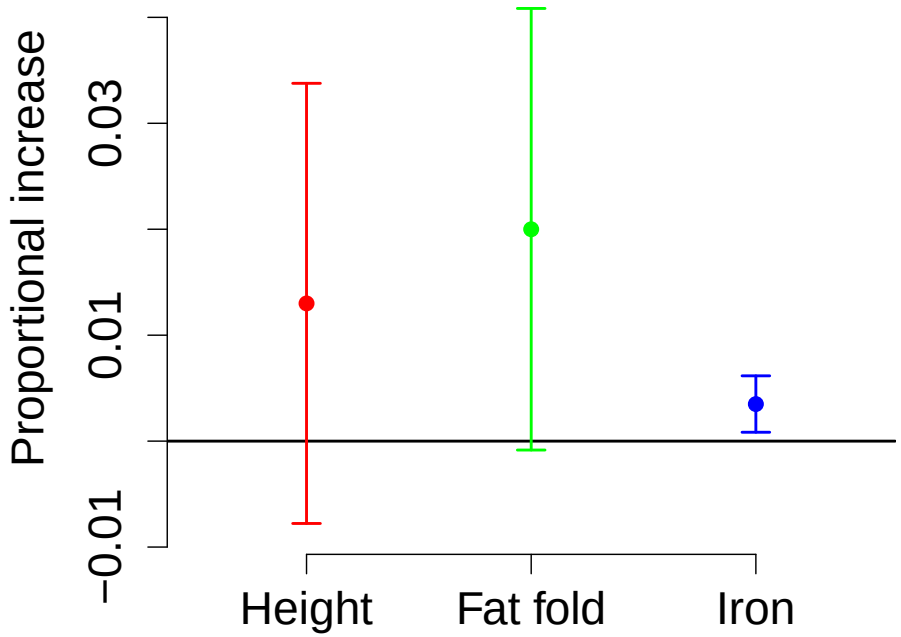
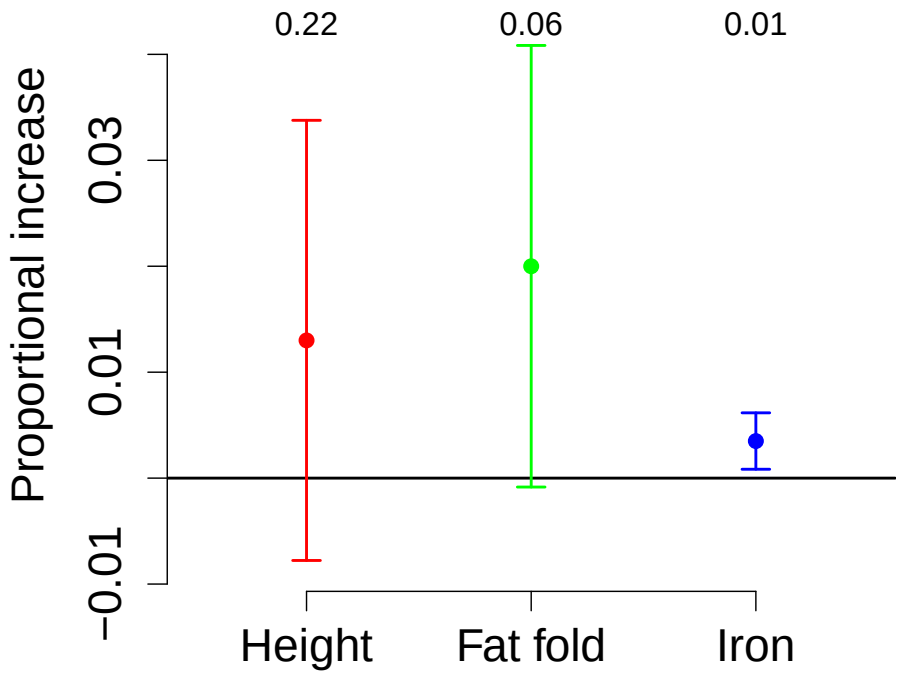



- What do these results mean?
- Which are significant?

# Confidence intervals and P values



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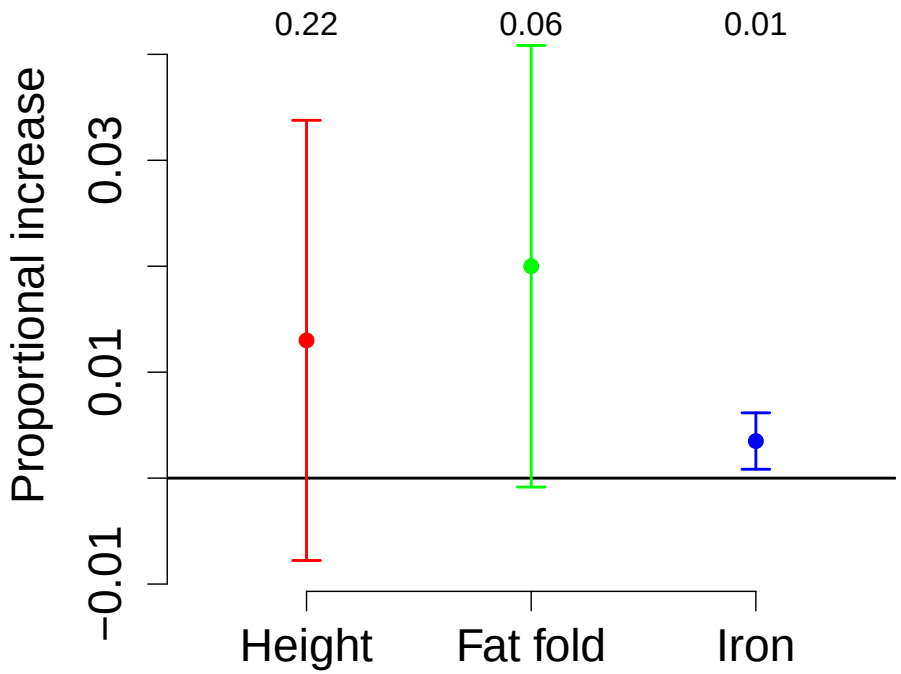


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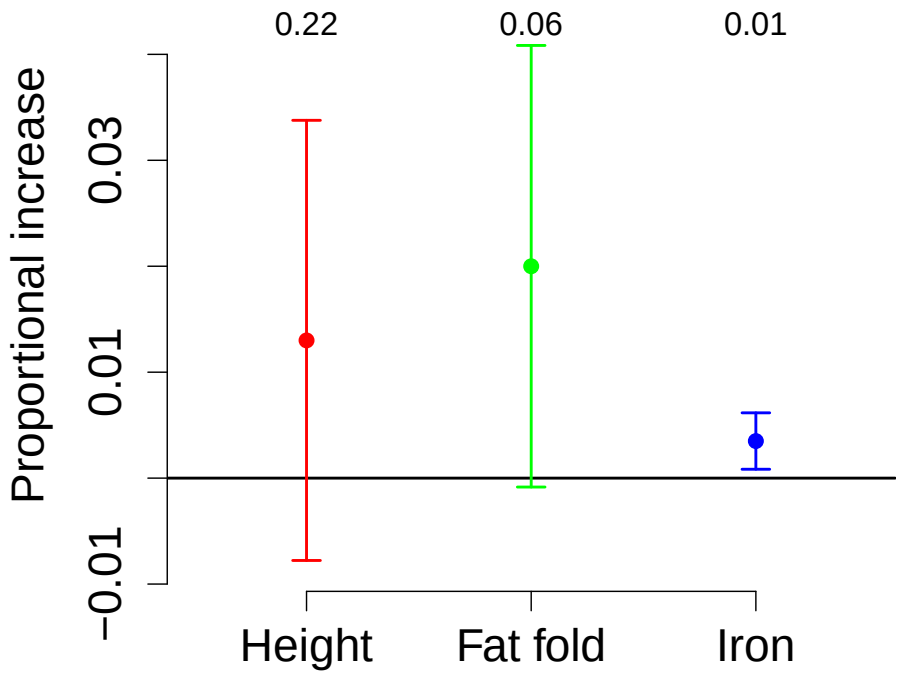


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- A low P value means we can

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  - It's usually the sign of some quantity, but doesn't need to be

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  - This doesn't happen in biology. There is always an effect.
- Type II (*False negative:*) concluding there is no effect when there really is
  - This *should* never happen, because we should never conclude there is no effect

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- This is basically power and validity analysis – you should do these hypothetical analyses *before* you collect data, not after

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- *Sign error:* if I think an effect is positive, when it's really negative (or vice versa)
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- Confidence intervals clarify all of this

# Low P values

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- If I have a low P value I can see something clearly

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- If I have a low P value I can see something clearly
- But it's usually better to focus on what I see than the P value

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- High P values should *not* be used to advance your conclusion

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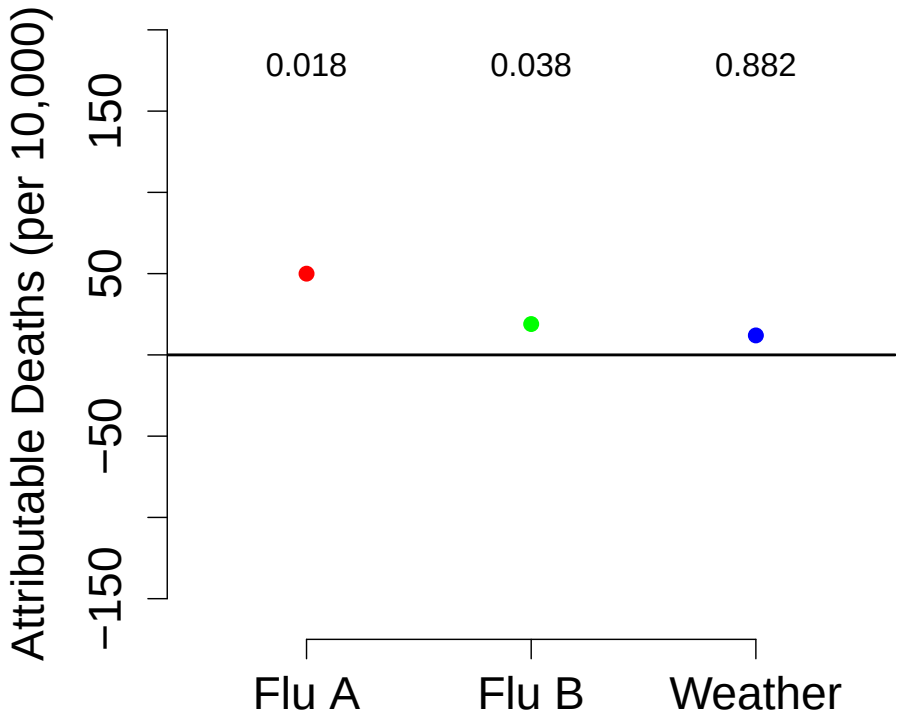
- Less model resolution
- A lower P value means that your evidence for difference is better
- A higher P value means that your evidence for similarity is better – or worse!

# Annualized flu deaths



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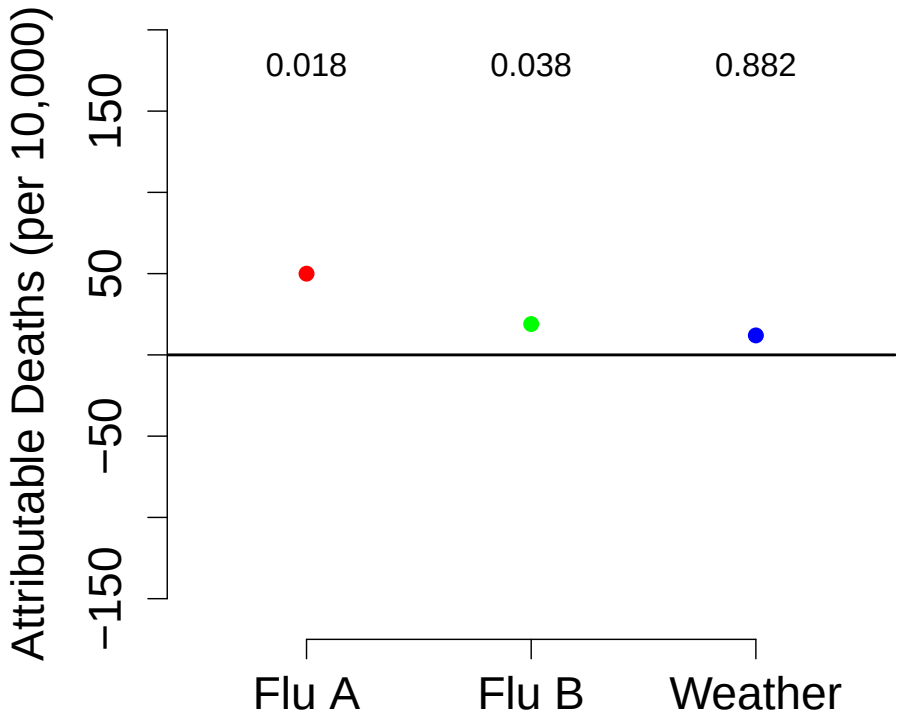
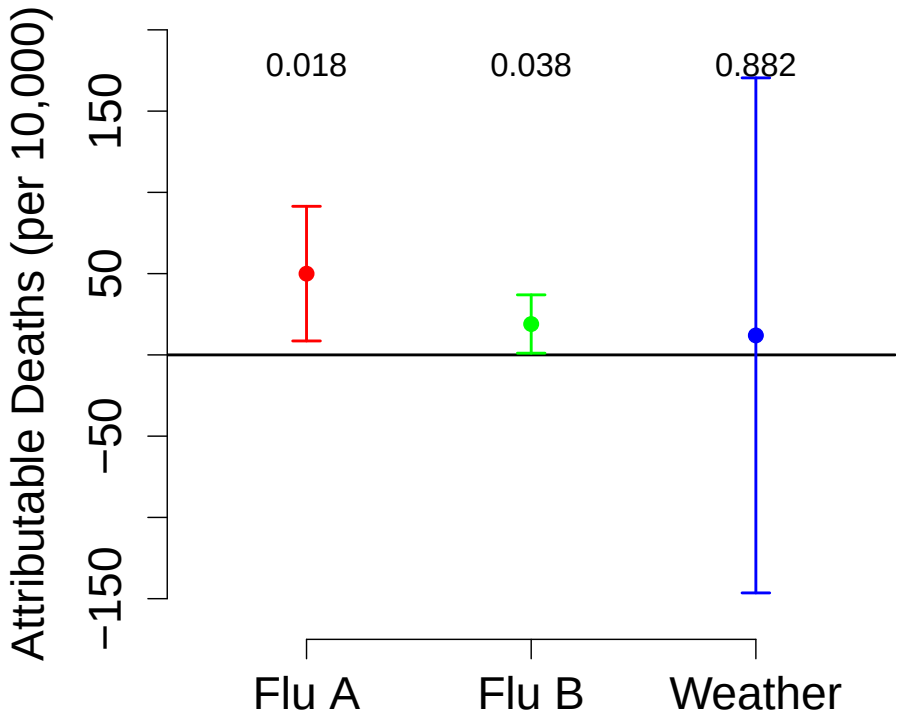



- Why is weather not causing deaths at this time scale?

# ... with confidence intervals



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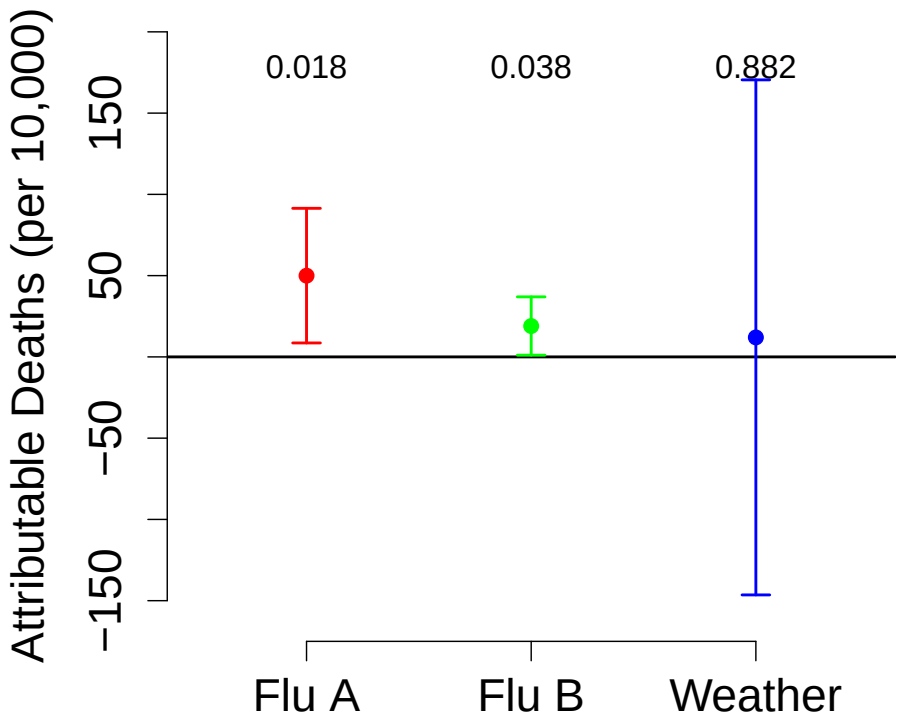


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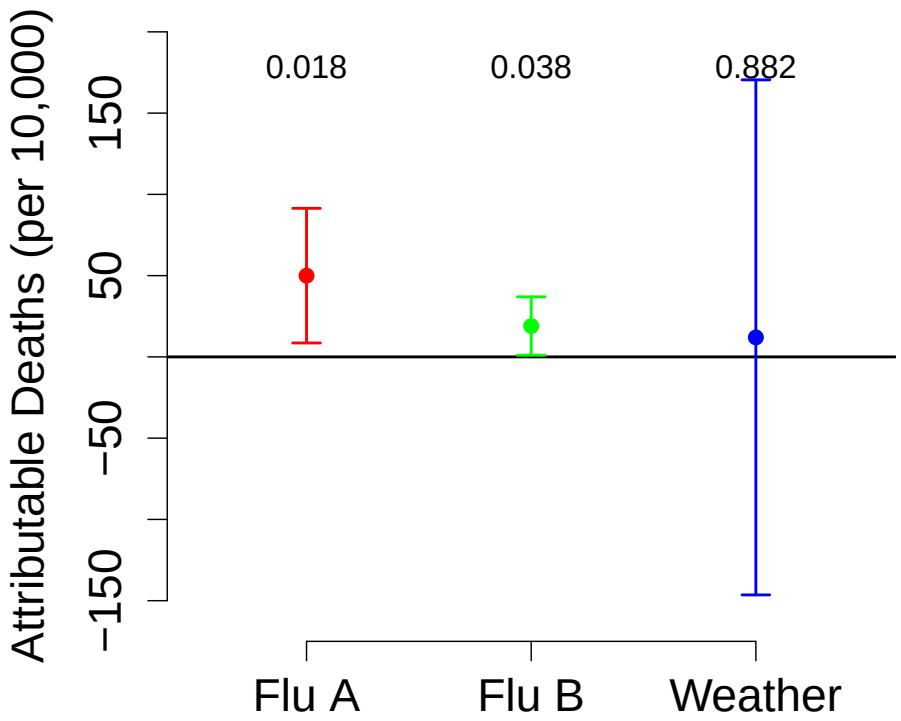


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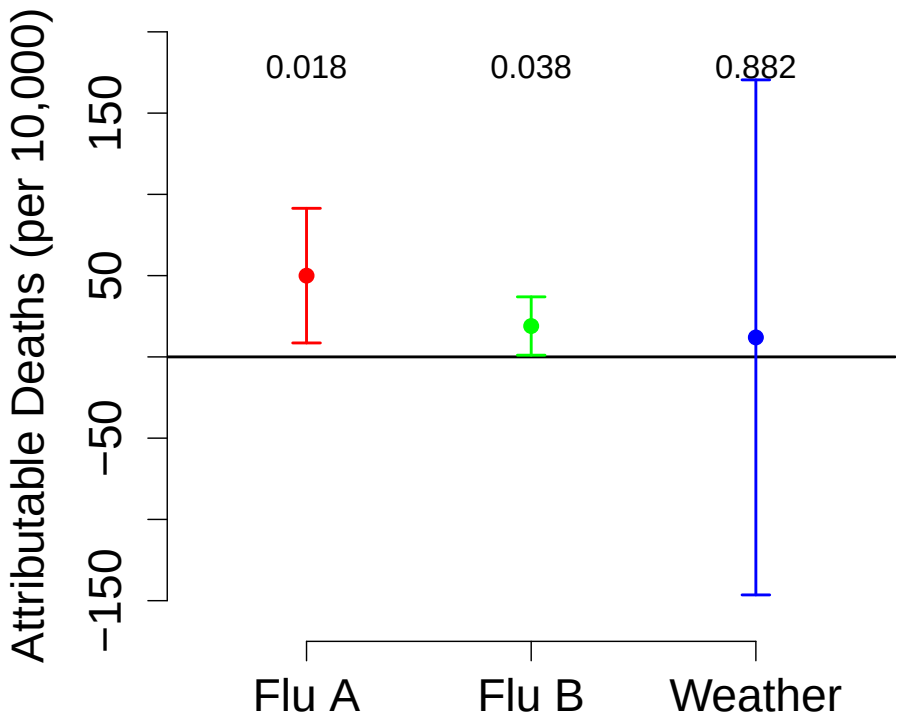


- Never say: A is significant and B isn't, so $A > B$
- Instead: Construct a statistic for the hypothesis $A > B$
  - Sometimes difficult, but you still have to do it

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  - Estimate a difference, or an interaction

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- This means we need a standard for what we mean by small

# Flu masks


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- They would like to switch to simpler masks, if those will do the job
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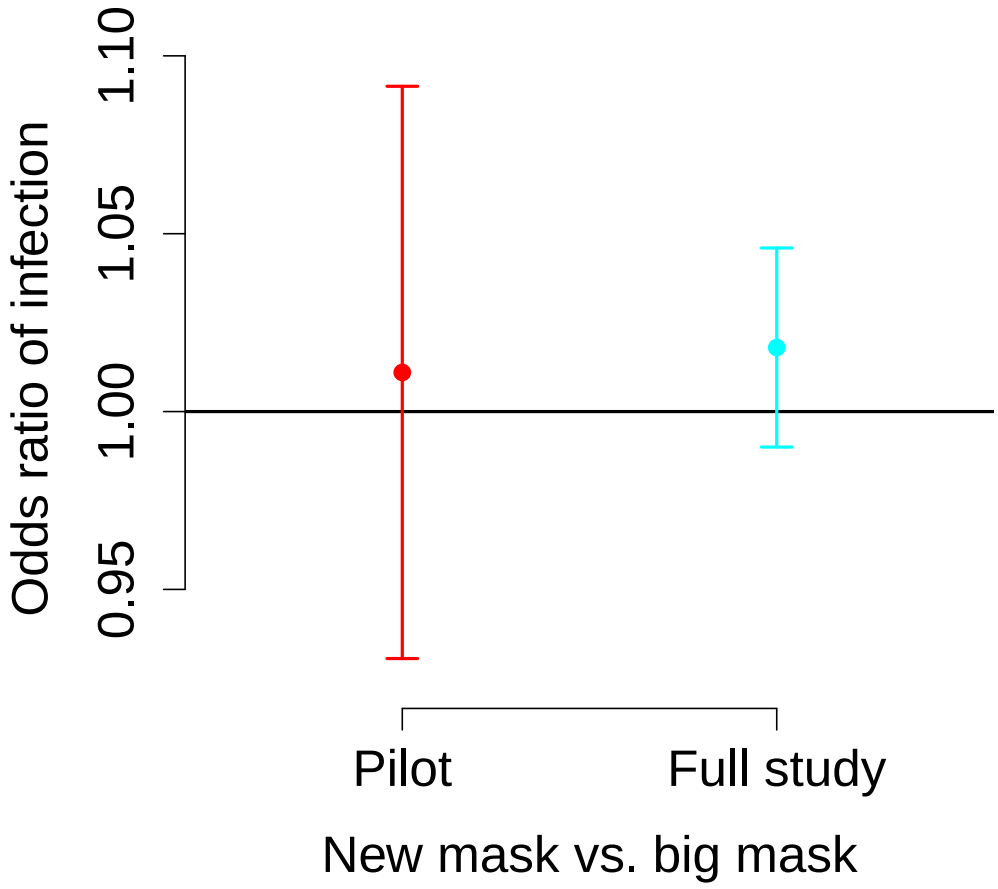
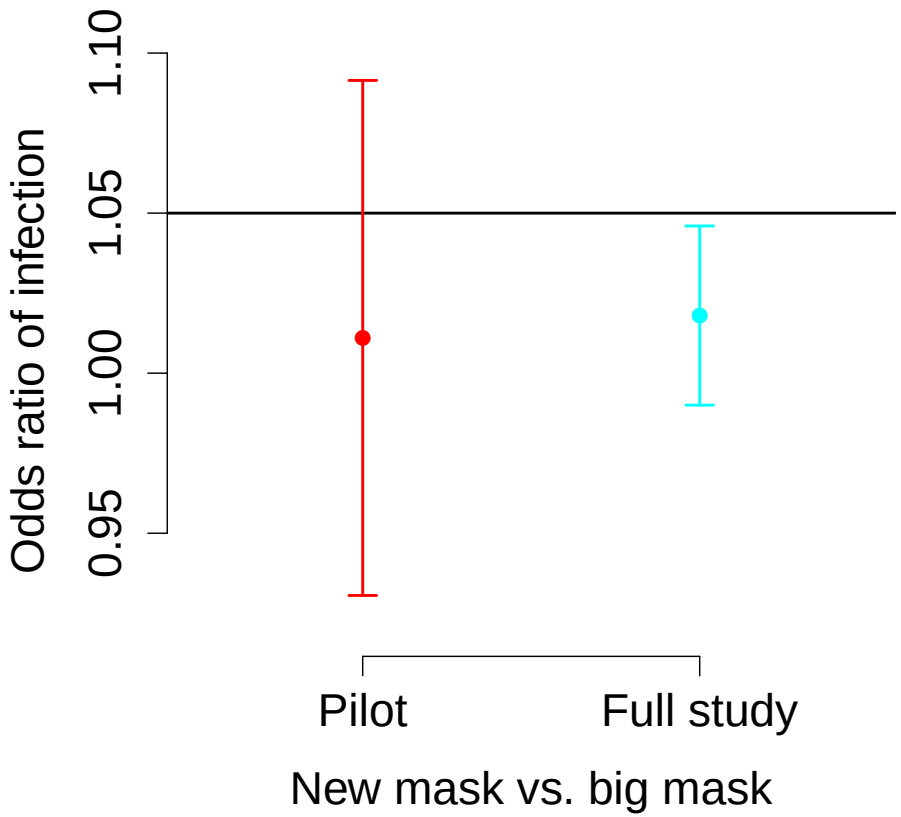
  - Use a confidence interval
  - Decide how big a level is acceptable, and construct a P value for the hypothesis that this level is excluded!

# Study results



# Non-inferiority trial



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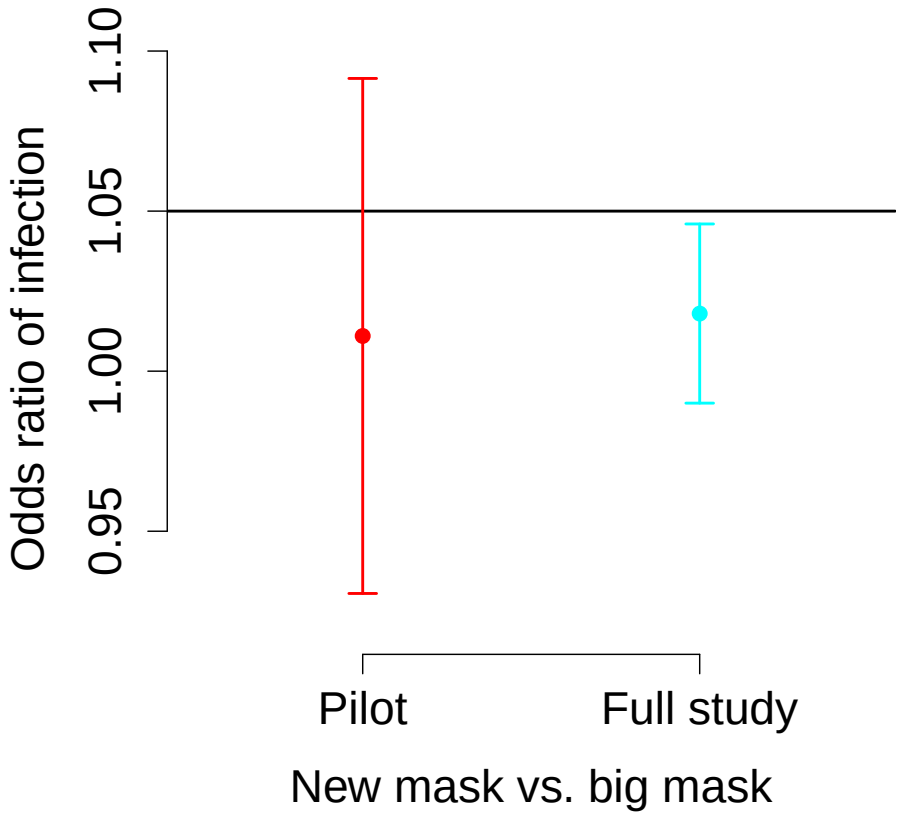


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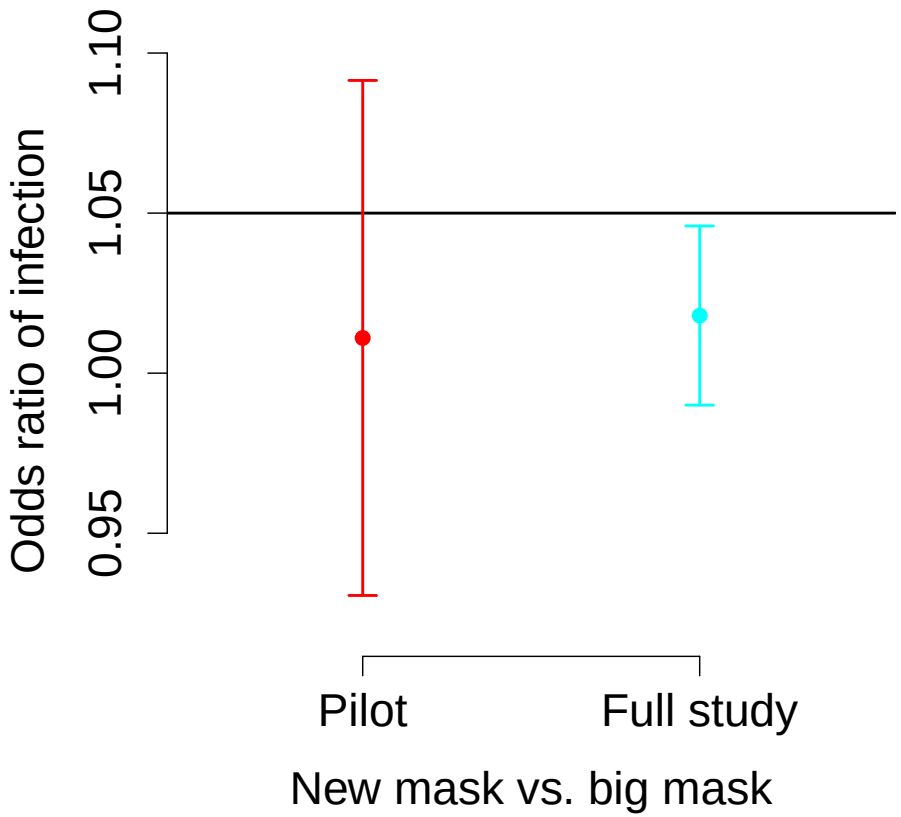


- Is the new mask "good enough"?
- What's our standard for that?

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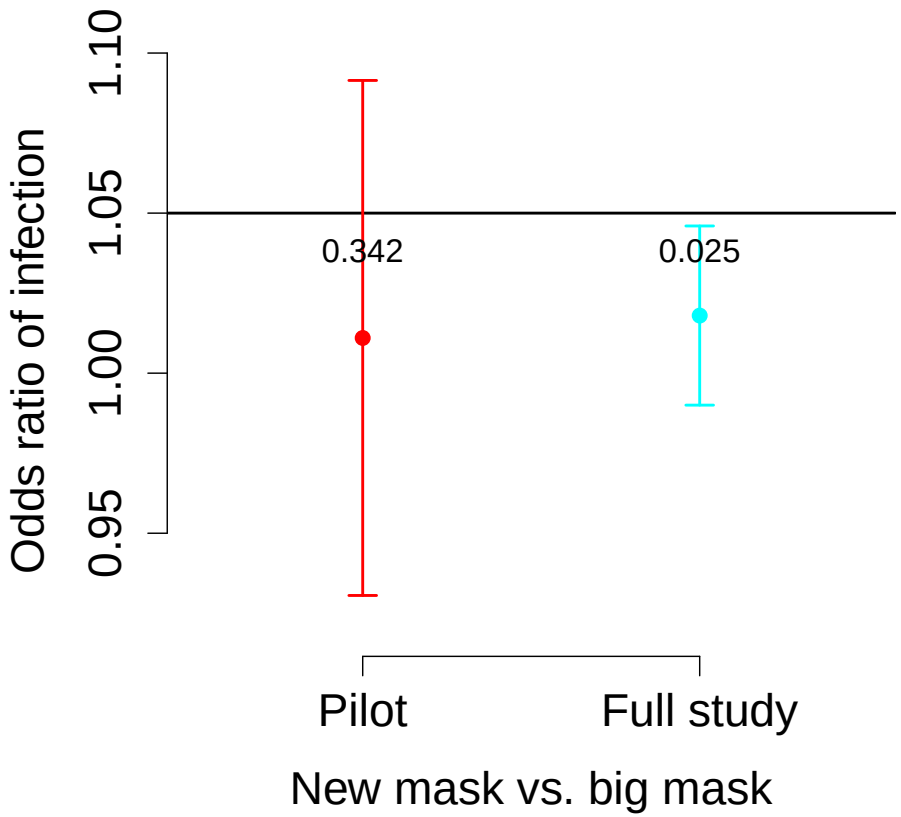

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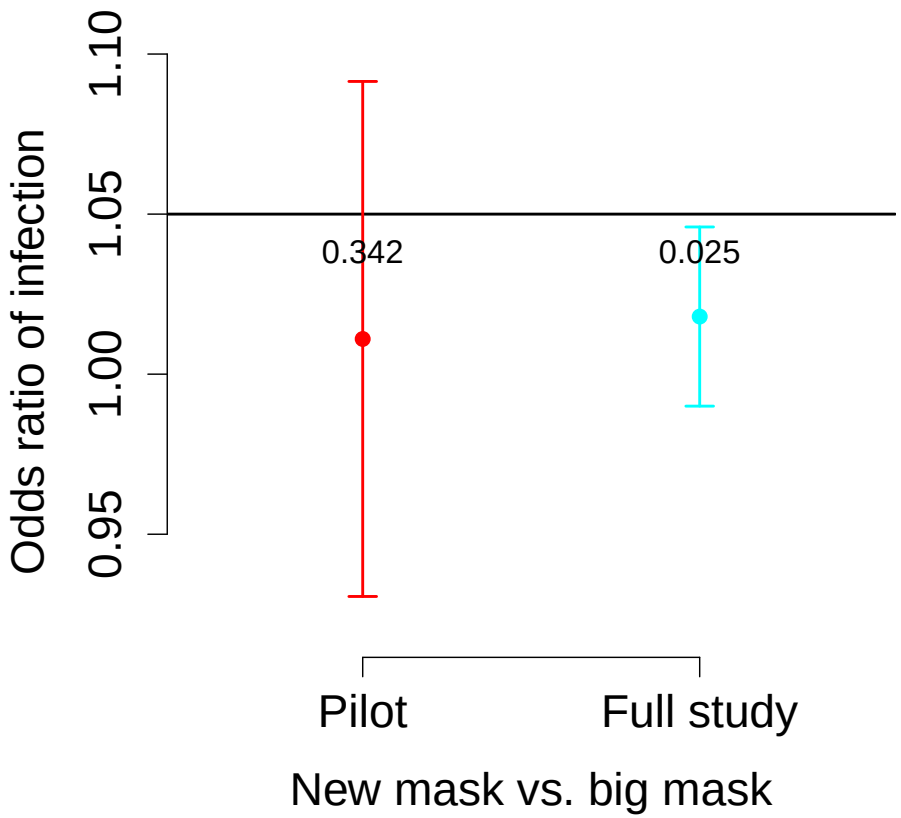


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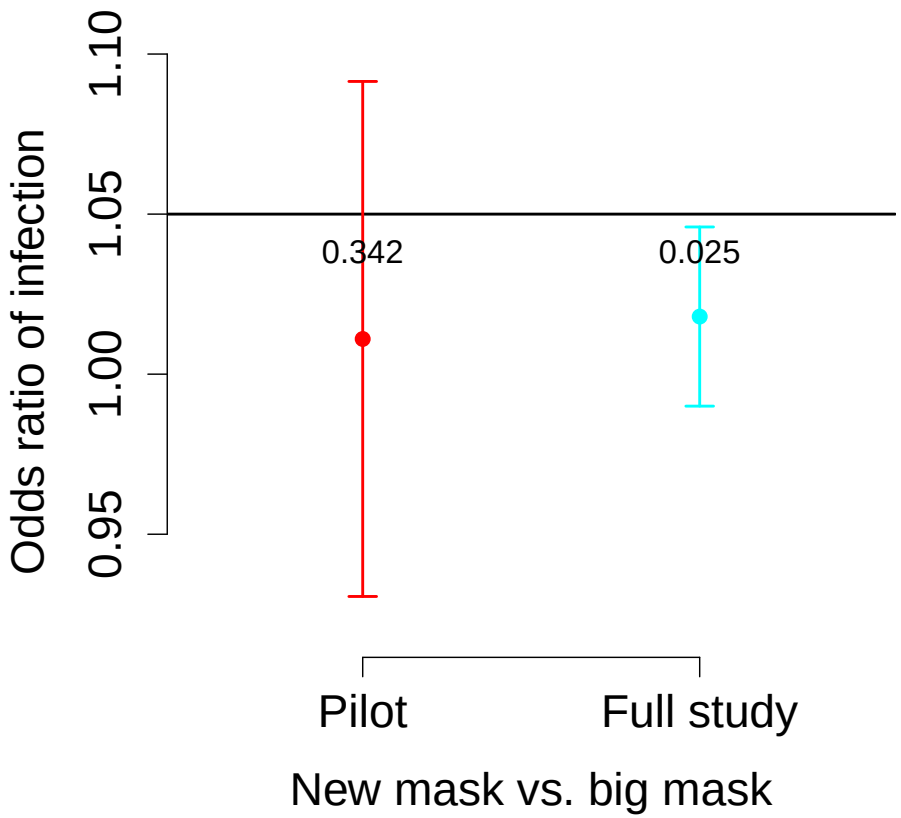


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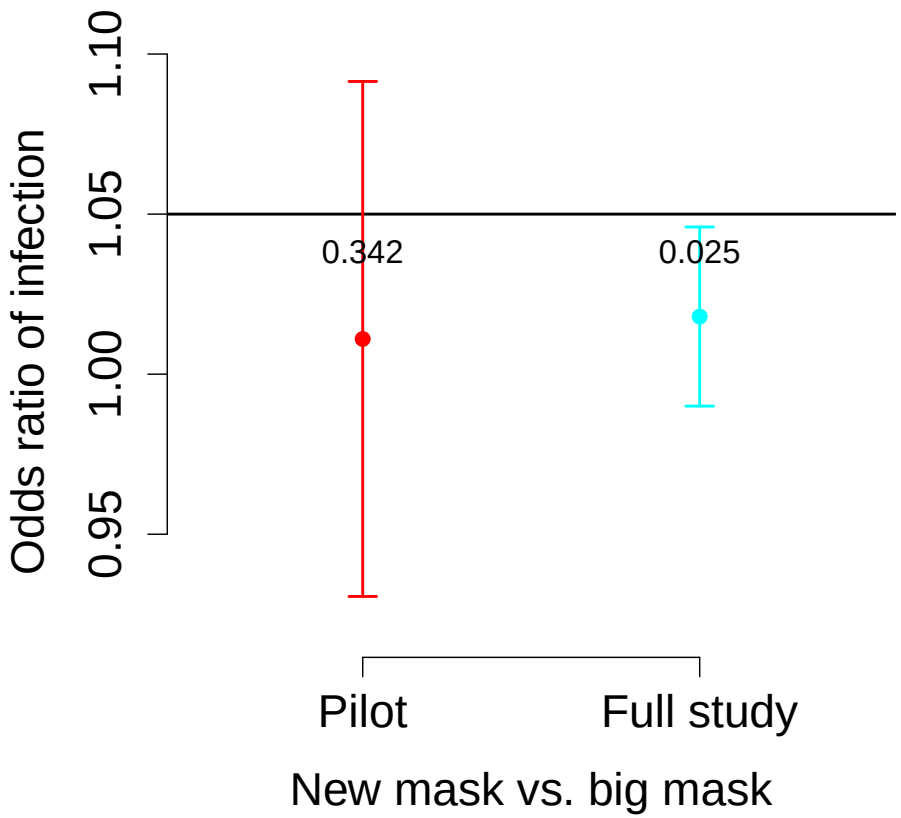


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- The right statistic is the thing whose sign we want to know:
  - The difference between the observed effect and the standard we chose

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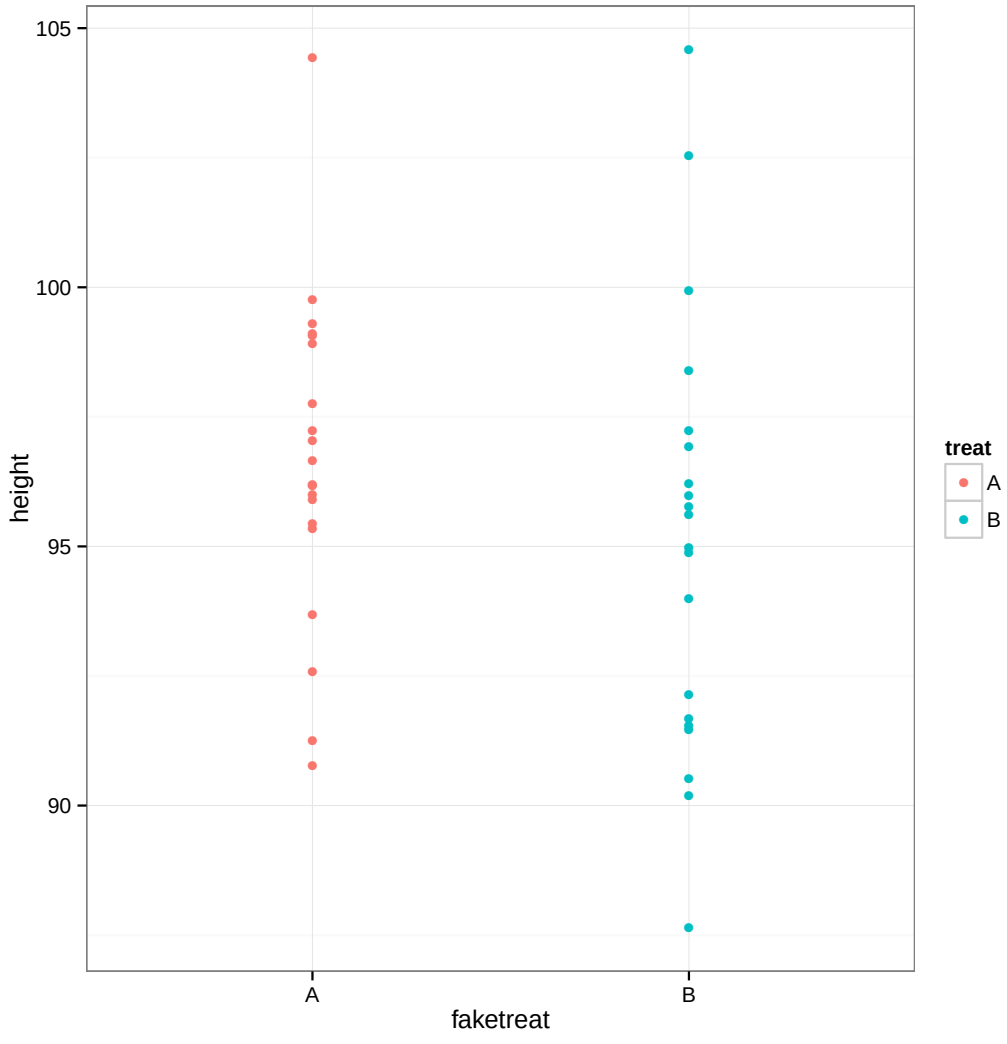
- Test whether the effect you see could be due to chance
  - What is the probability of seeing exactly a 1.52 cm difference in average heights?
- Test whether the effect you see *or a larger effect* could be due to chance
  - This probability is the P value

# Height measurements



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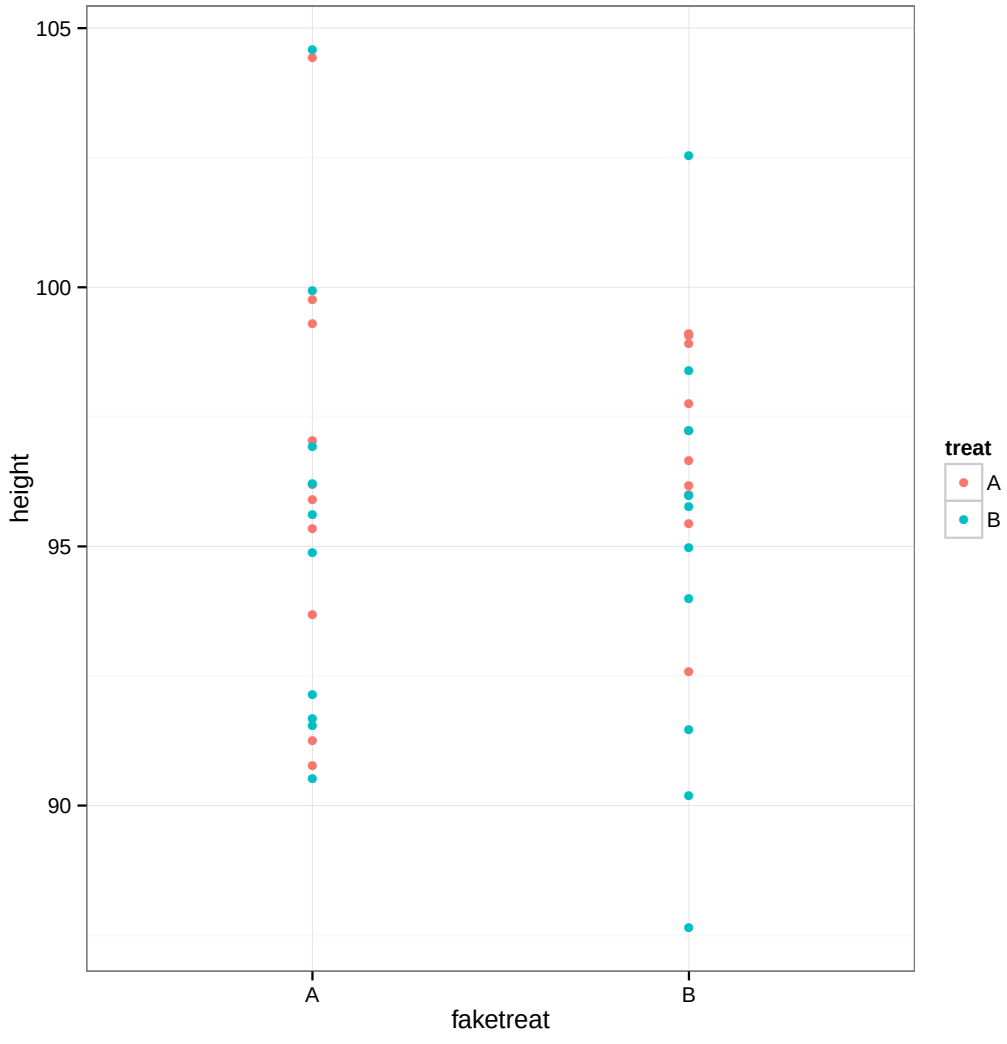

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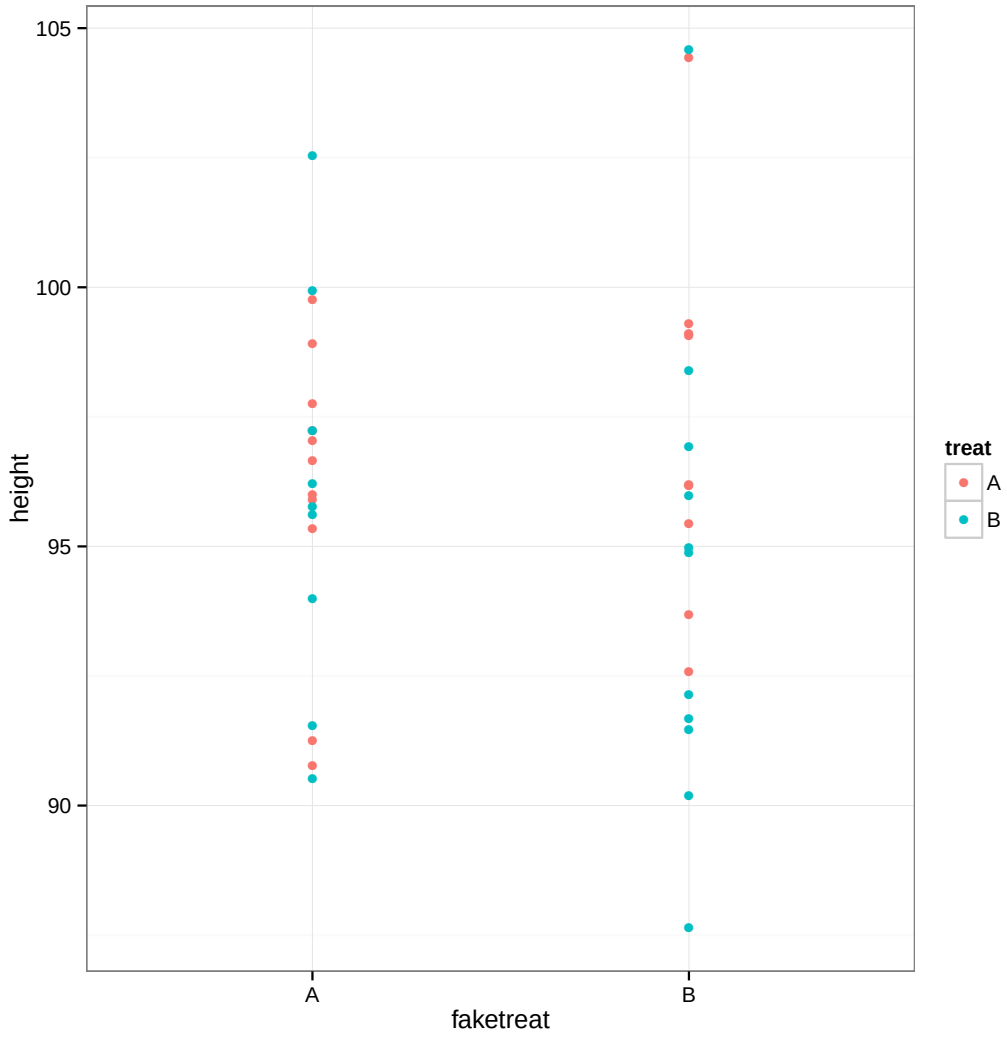

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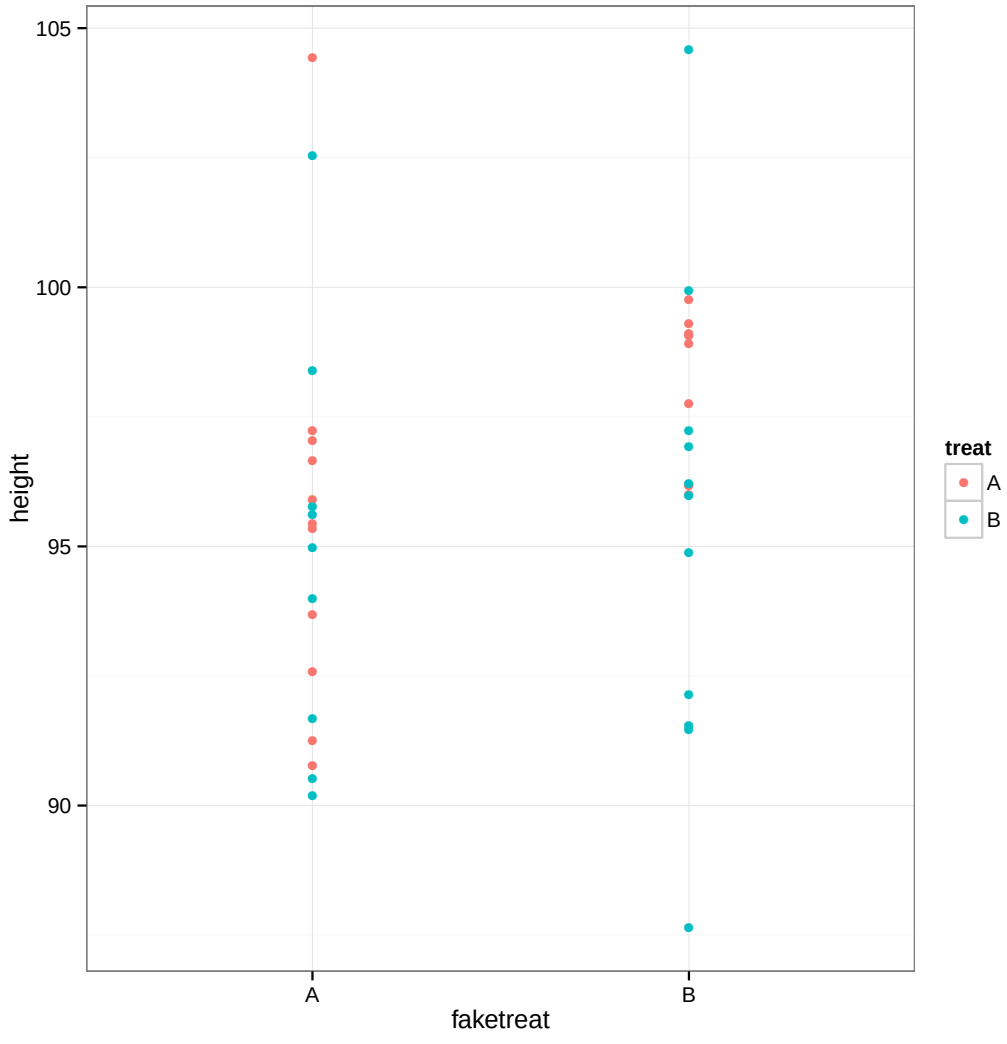

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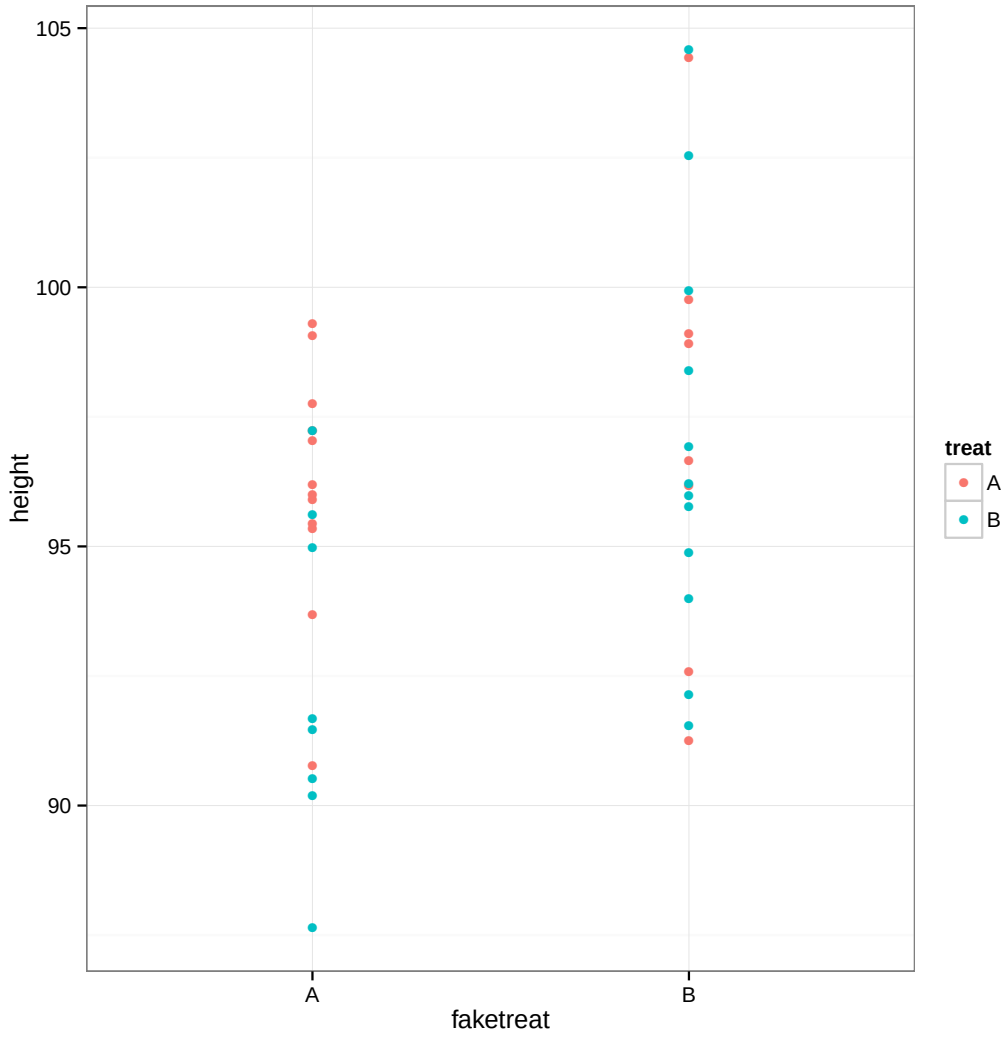
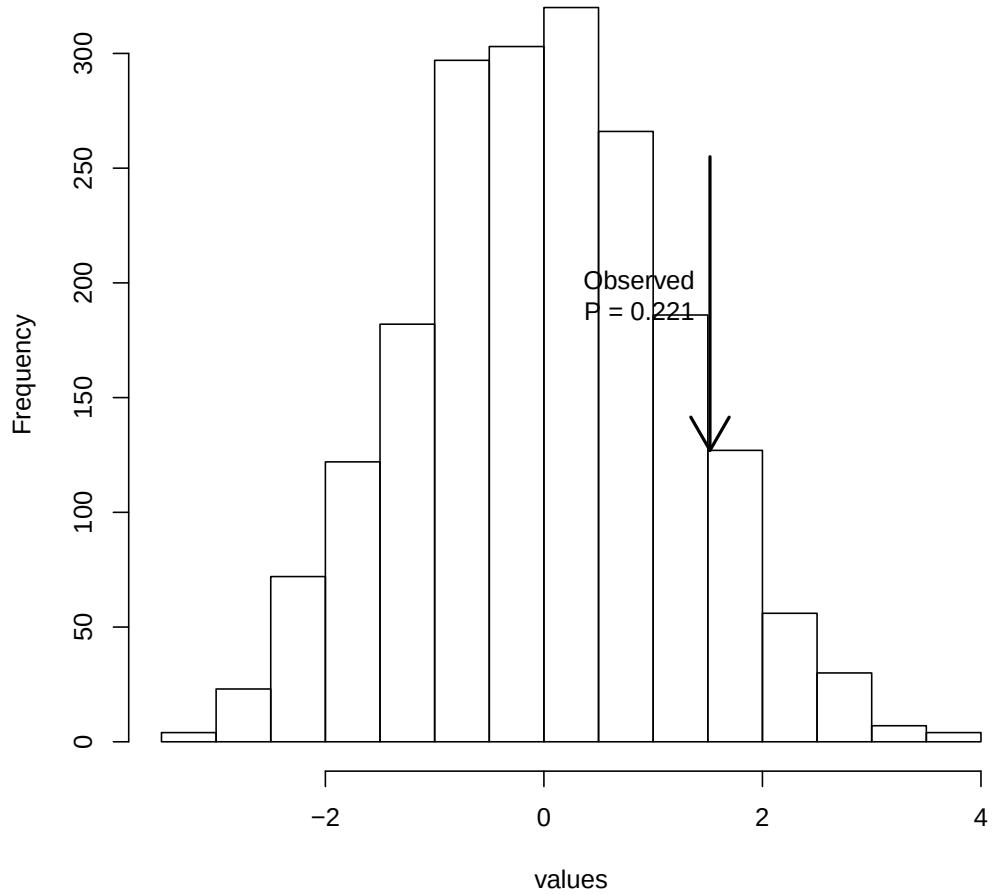


# The null distribution



# Outline

# Bayesian paradigm

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- Make a complete model world

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- Use conditional probability to calculate the probability you want

# A powerful framework

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- With great power comes great responsibility

# Bayesian inference

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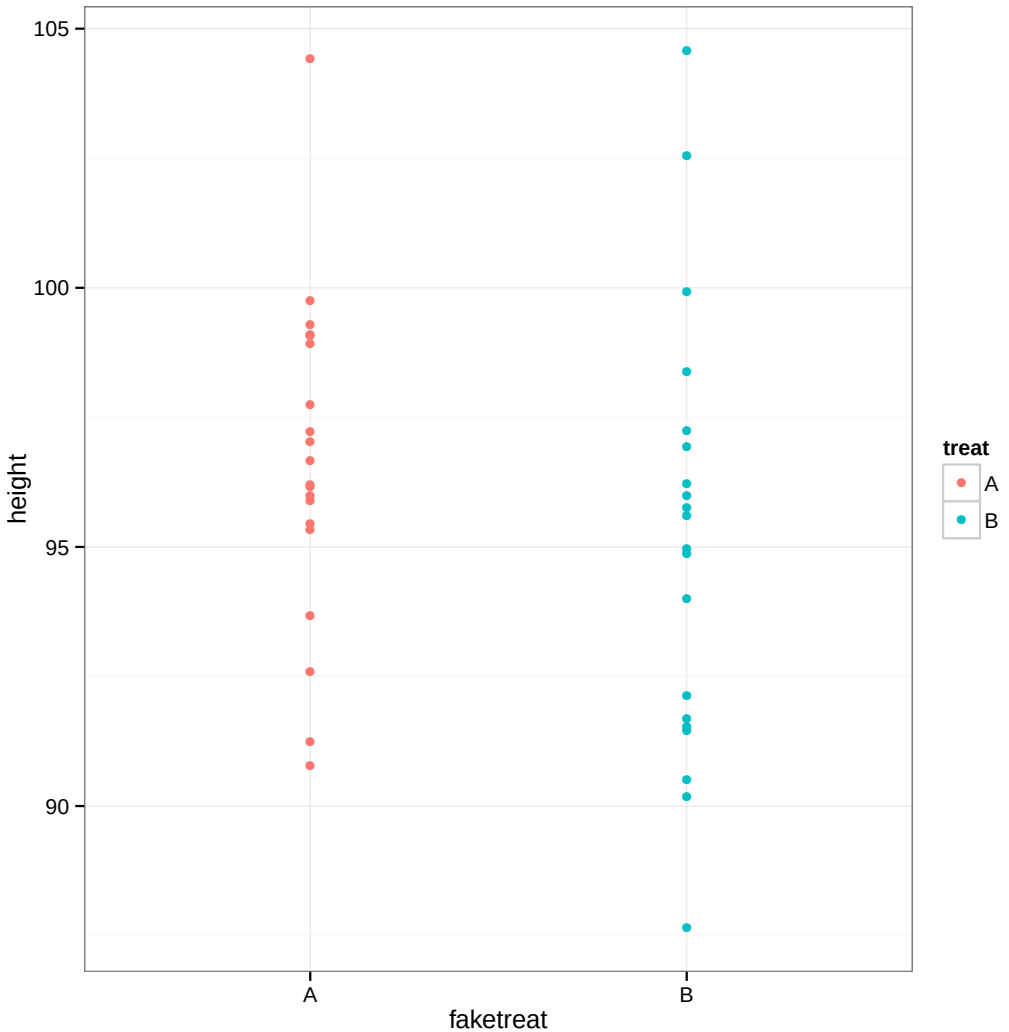
  - Requires *prior* distributions describing the assumed likelihood of parameters before these observations are made
  - Use Bayes theorem to calculate posterior distribution – likelihood after taking data into account

# Vitamin A study



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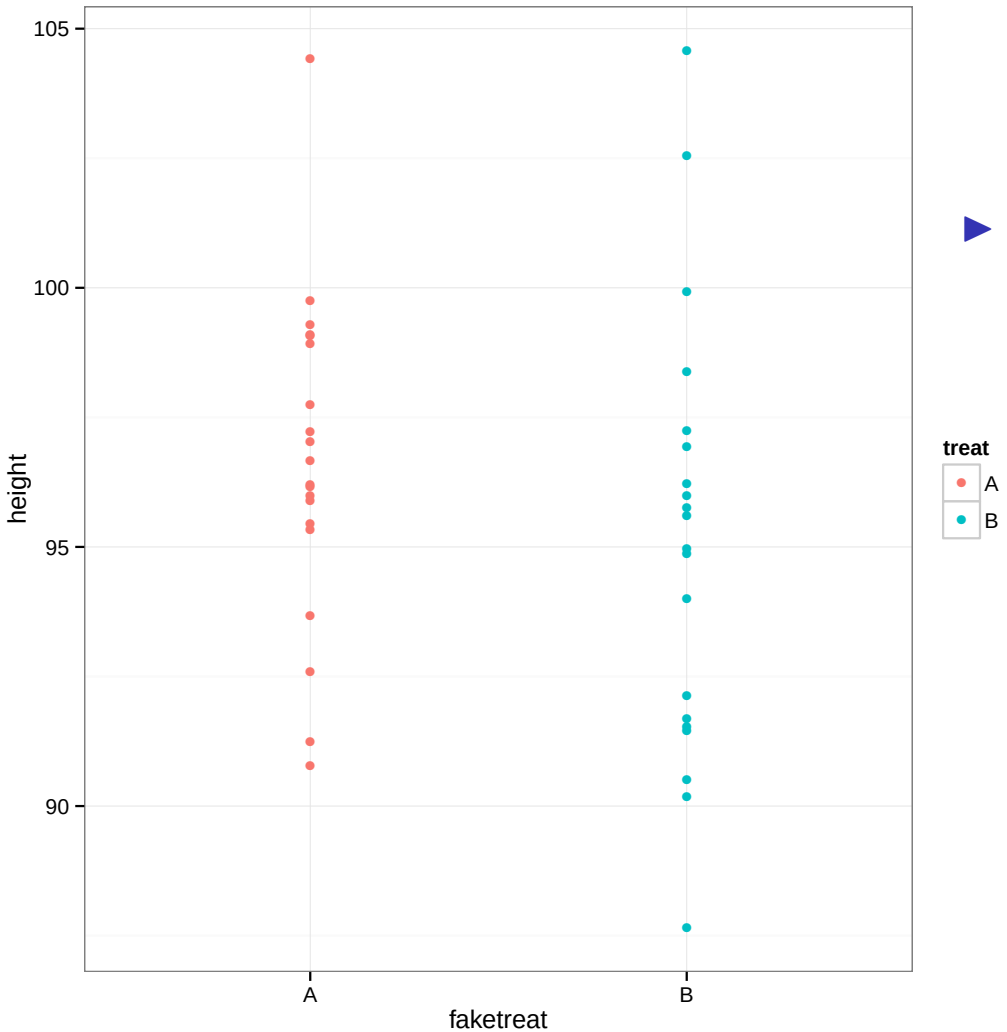


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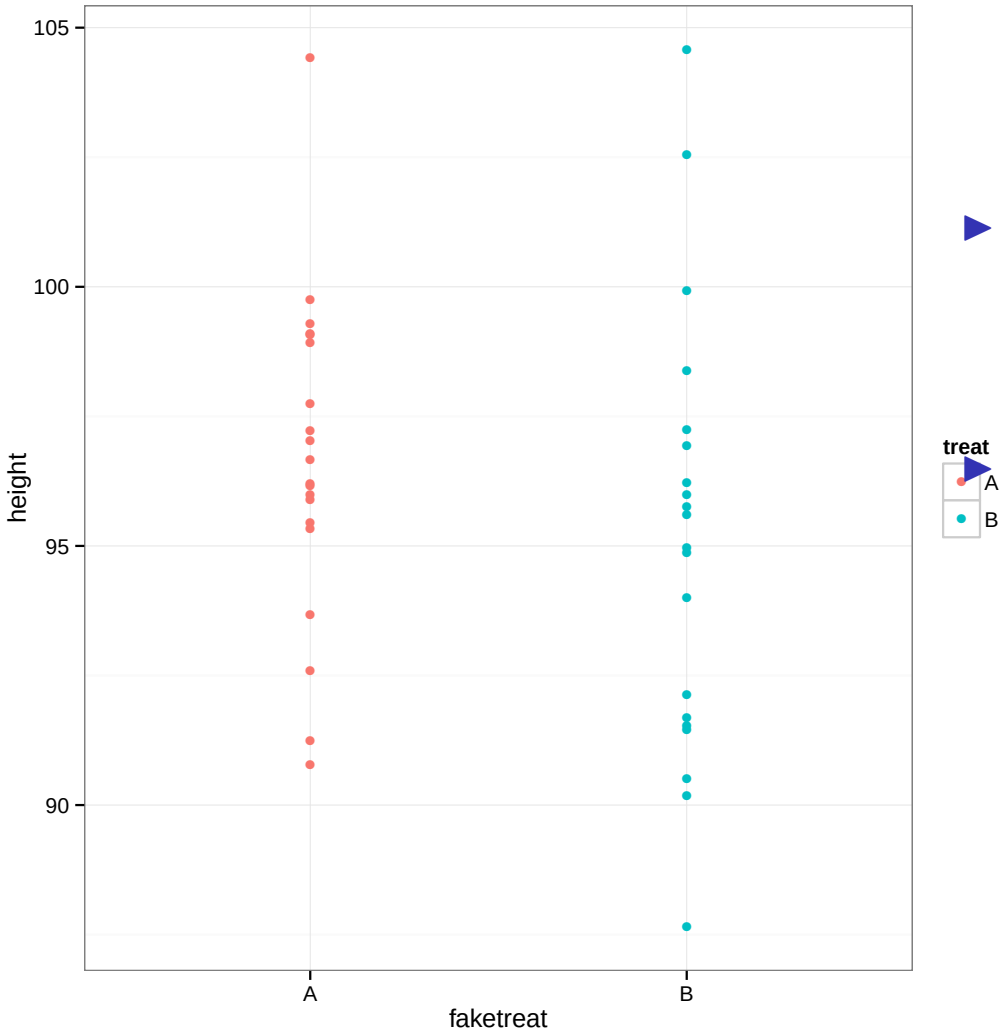


- A frequentist can do a clear analysis right away
- A Bayesian needs a ton of assumptions – will try to make “uninformative” assumptions

# Cape Town weather

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- Bayesian: what's my model world? What is my prior belief about weather-weekday interactions.

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  - What do you tell them?

# Outline

# Your philosophy

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  - Be honest: it's harder than it sounds.

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- Reference: “Garden of forking paths"