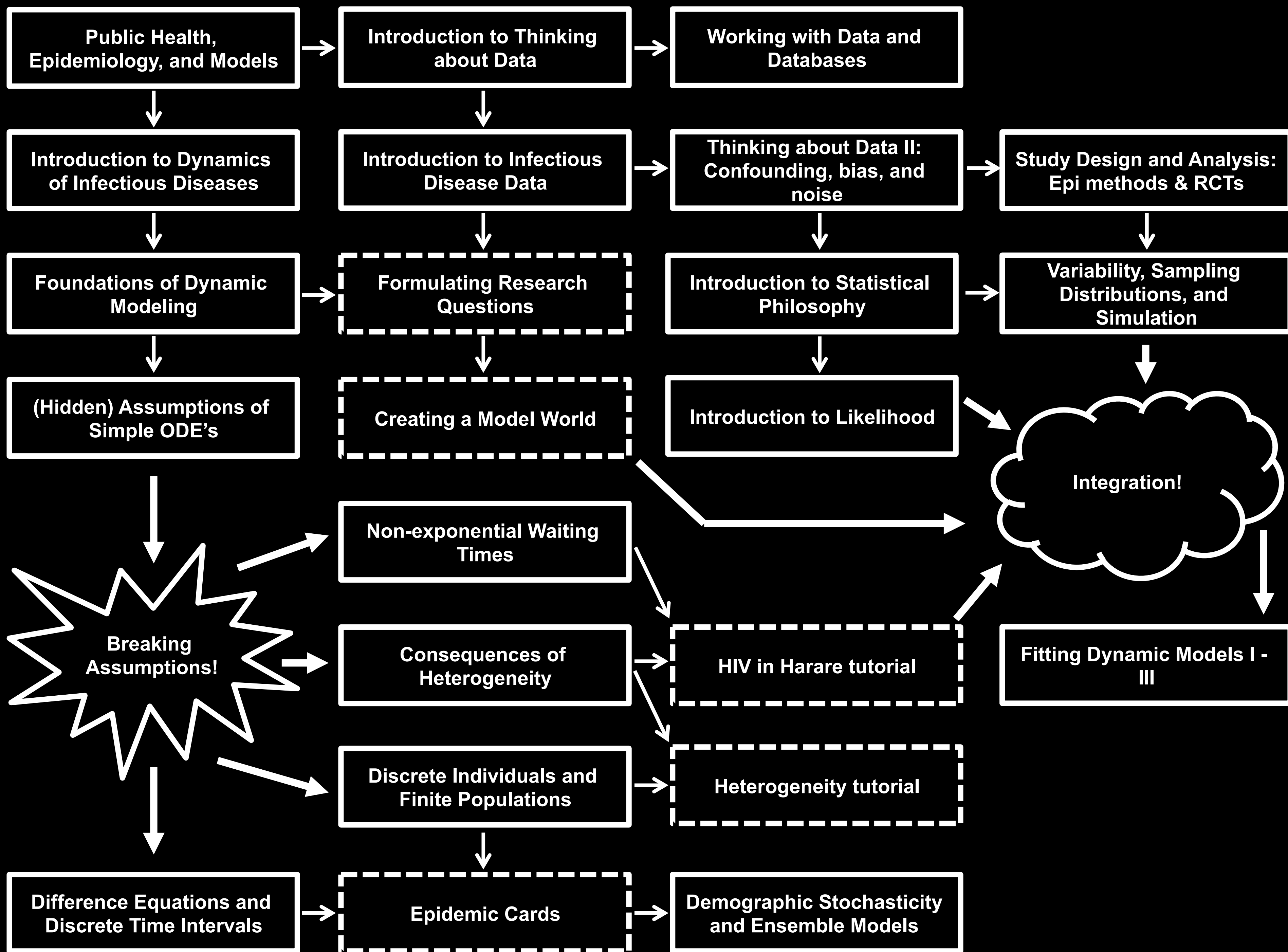

Public Health, Epidemiology, and Models
Introduction to Thinking about Data
Working with Data and Databases
Introduction to Dynamics of Infectious Diseases
Introduction to Infectious Disease Data
Thinking about Data II: Confounding, bias, and noise
Study Design and Analysis: Epi methods & RCTs
Foundations of Dynamic Modeling
Formulating Research Questions
Introduction to Statistical Philosophy
Variability, Sampling Distributions, and Simulation
(Hidden) Assumptions of Simple ODE's
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Integration!
Non-exponential Waiting Times
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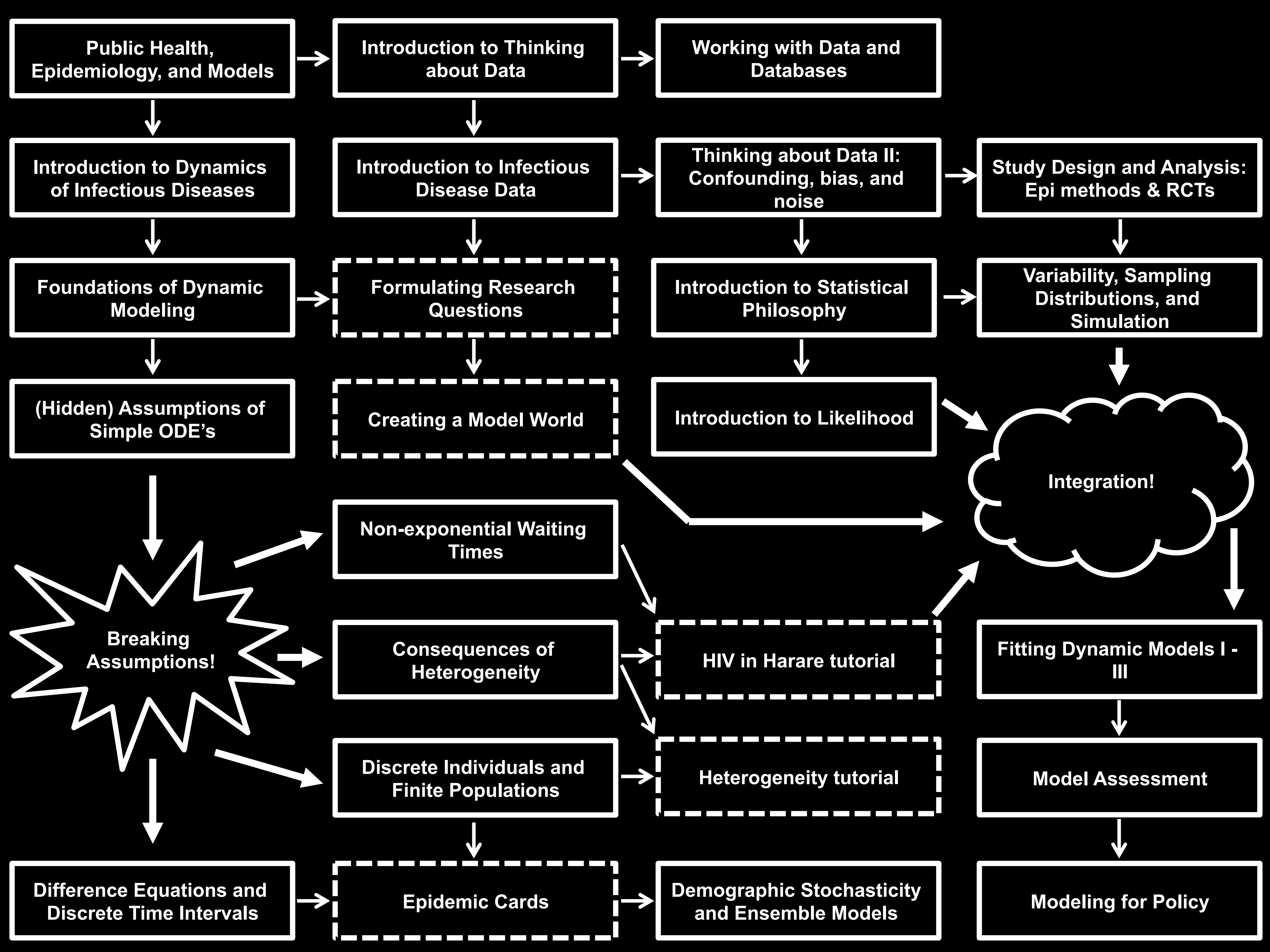
Demographic Stochasticity and Ensemble Models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Modeling for Policy

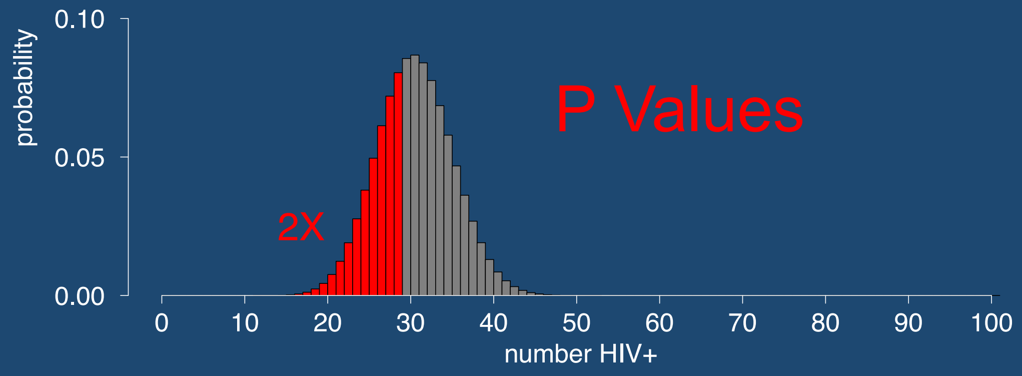
Likelihood fitting and dynamic models: Part 1

Steve Bellan, PhD, MPH
Center for Computational Biology and Bioinformatics
The University of Texas at Austin

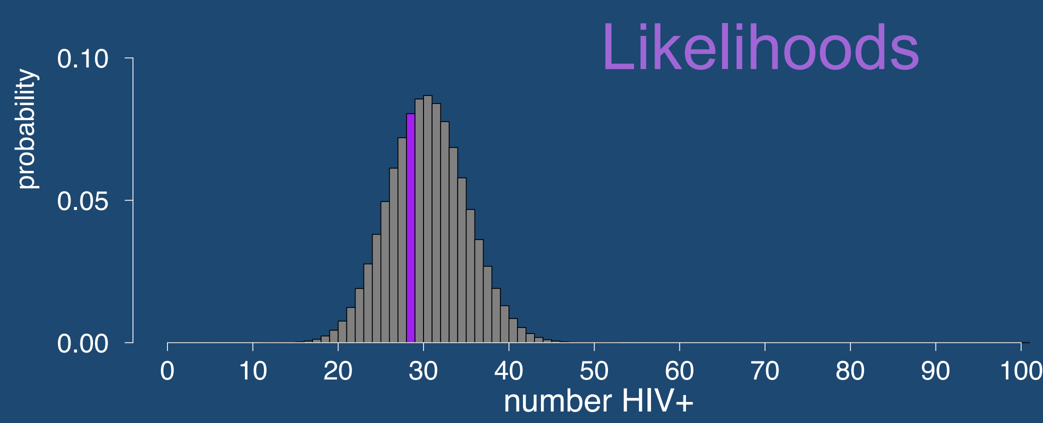
June 3, 2016

Clinic on the Meaningful Modeling of Epidemiological Data
ICI3D Program and AIMS - South Arica

Integration Is Hard



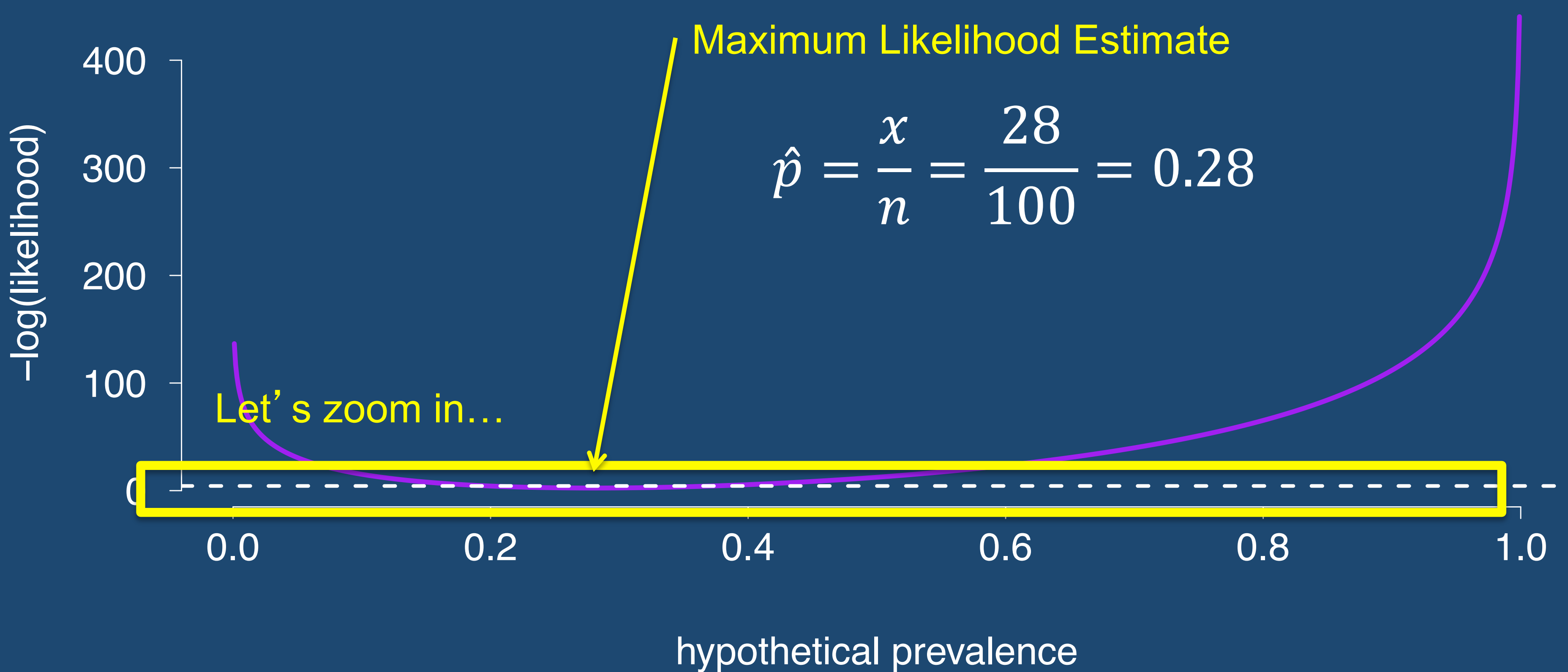
Do we want to sum or integrate the area under a curve?



Or just evaluate a function at one point?

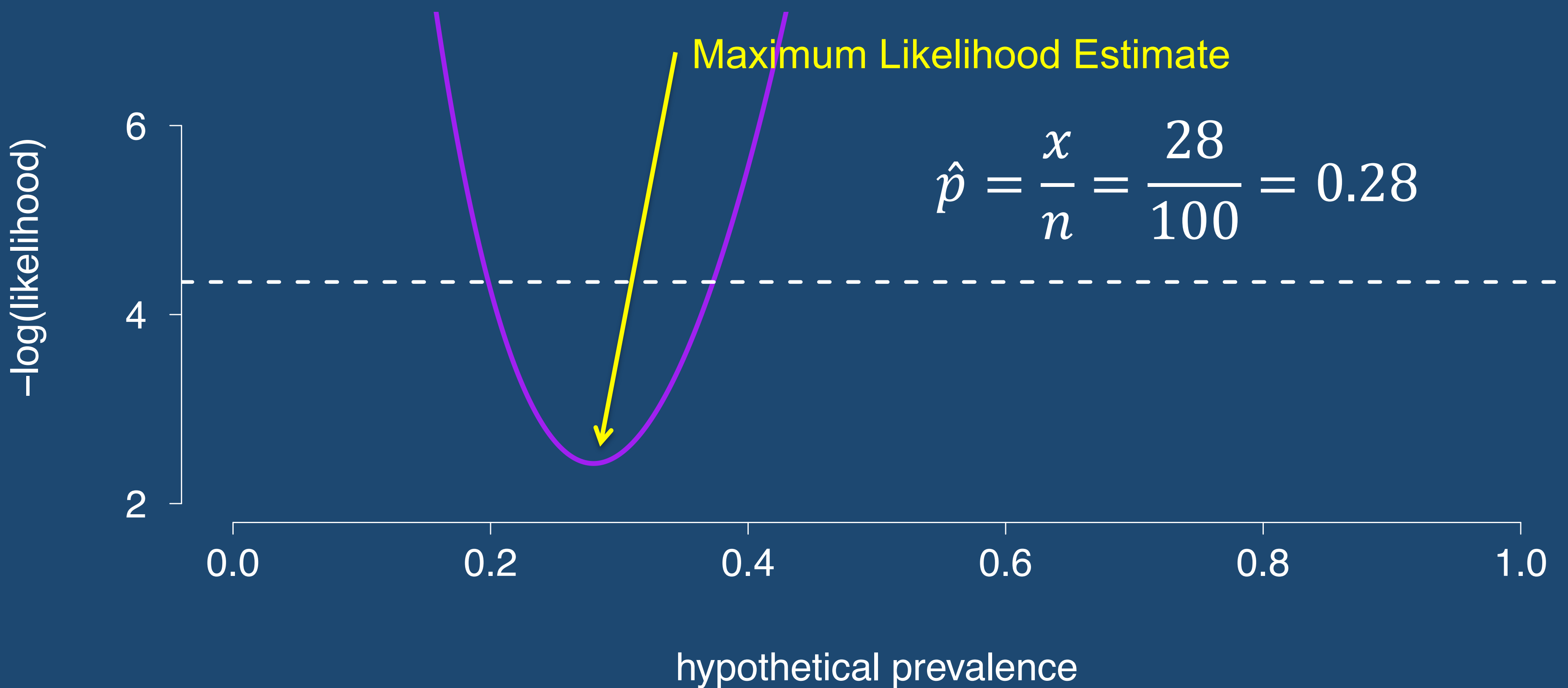
Building Confidence Intervals
Likelihood Ratio Test

we usually minimize the –log(likelihood)



Building Confidence Intervals

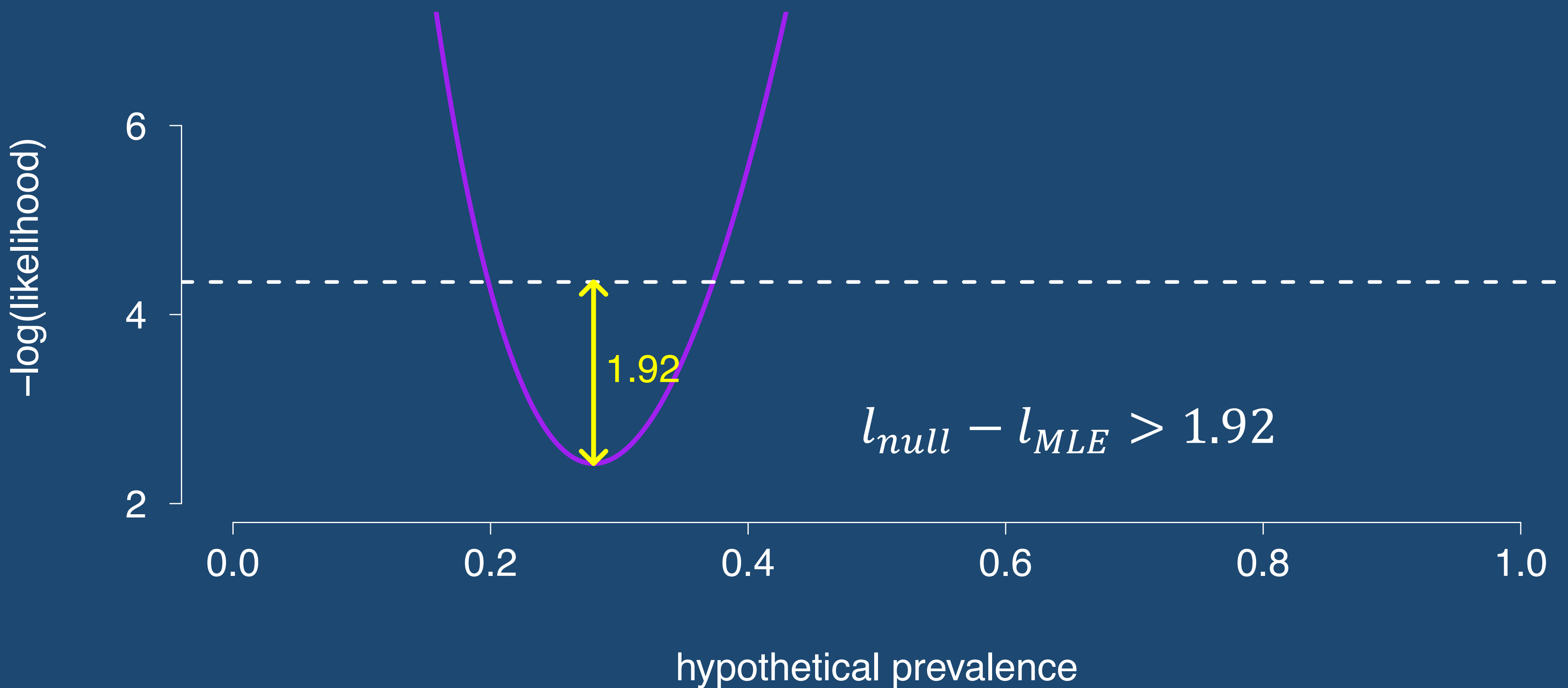
Likelihood Ratio Test



$$\hat{p} = \frac{x}{n} = \frac{28}{100} = 0.28$$

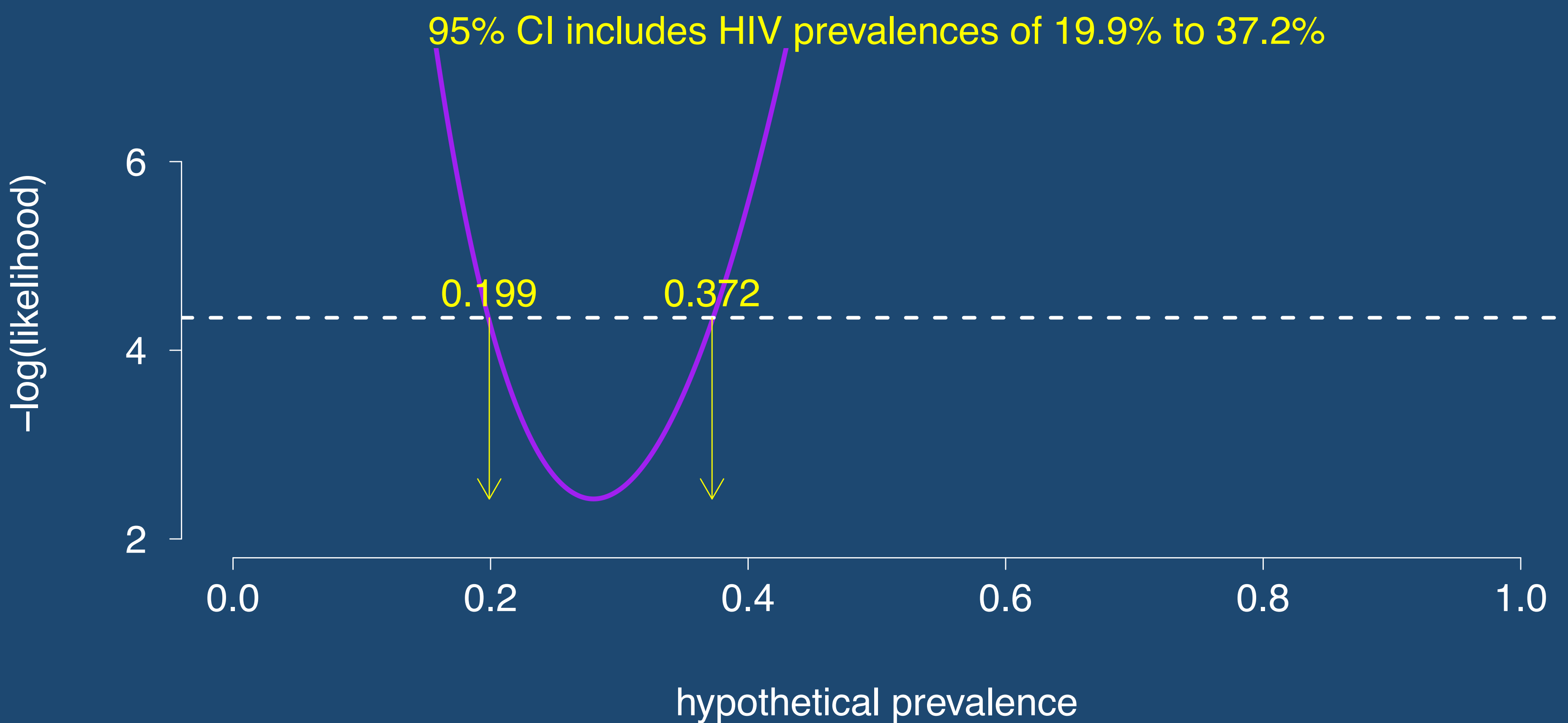
Building Confidence Intervals

Likelihood Ratio Test

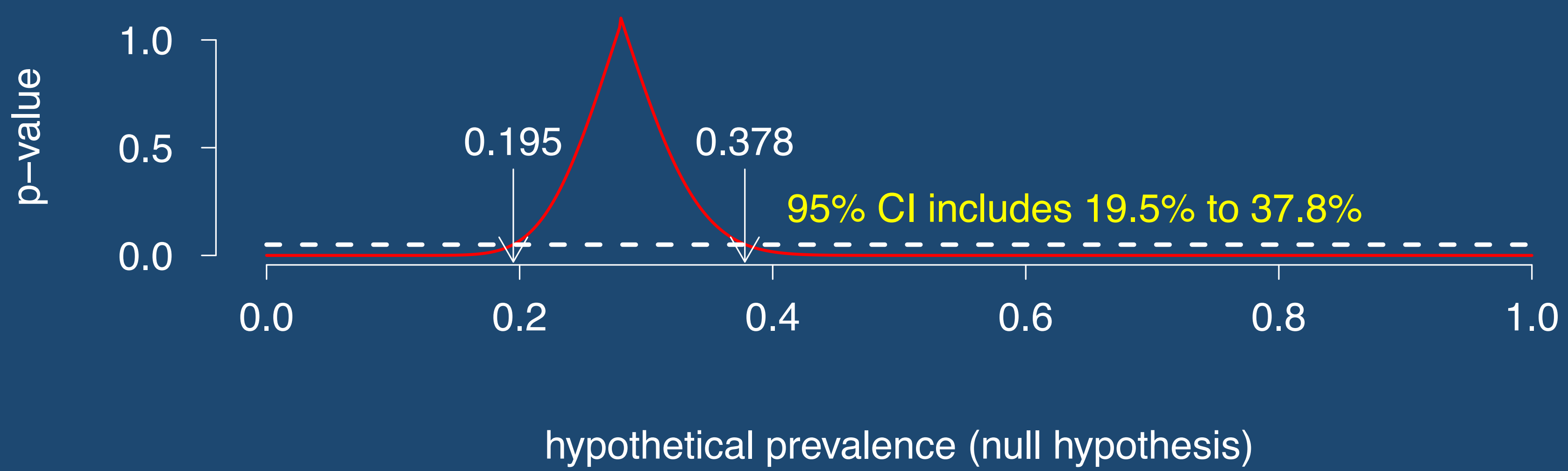


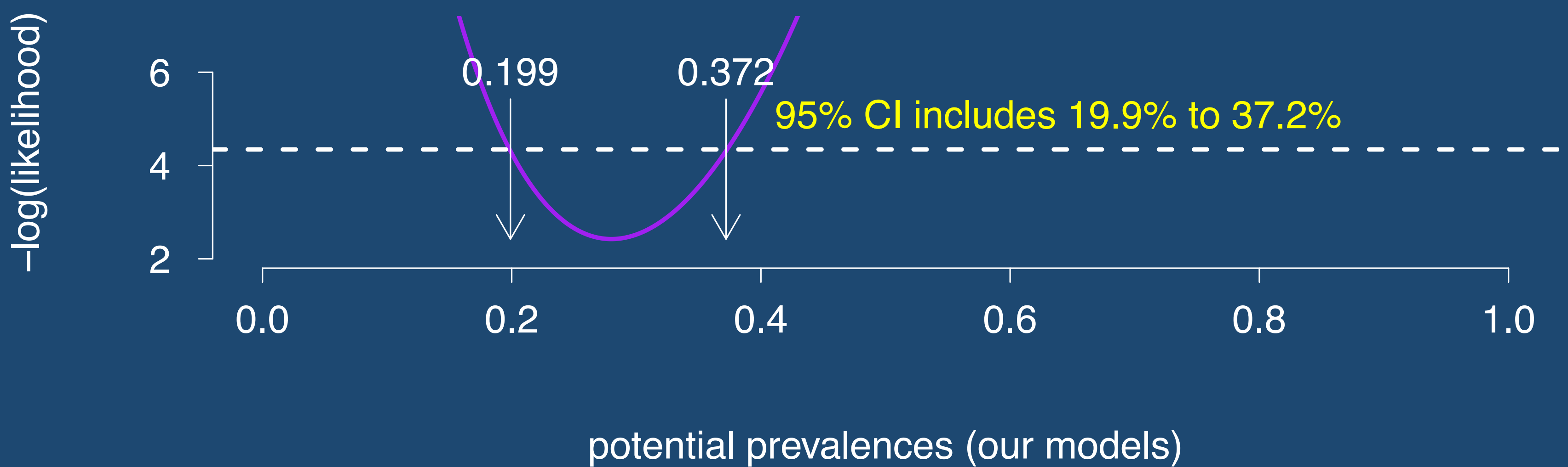
$$l_{null} - l_{MLE} > 1.92$$

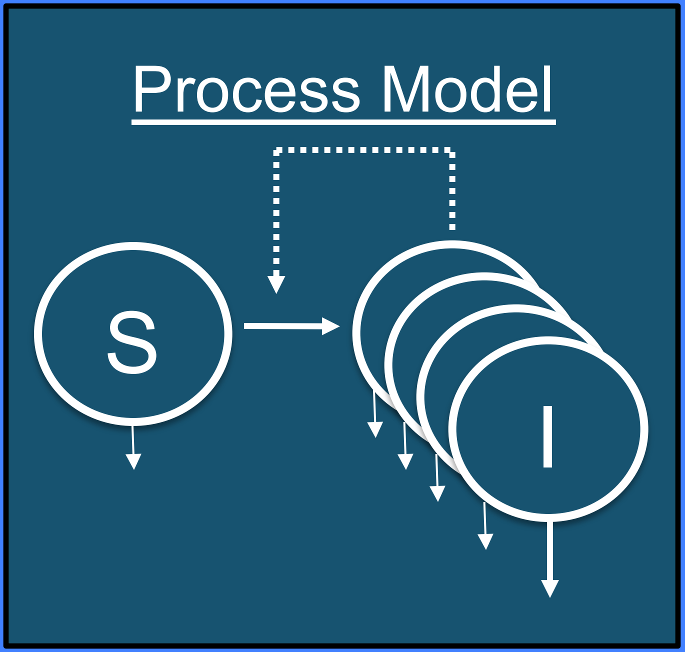
Building Confidence Intervals
Likelihood Ratio Test

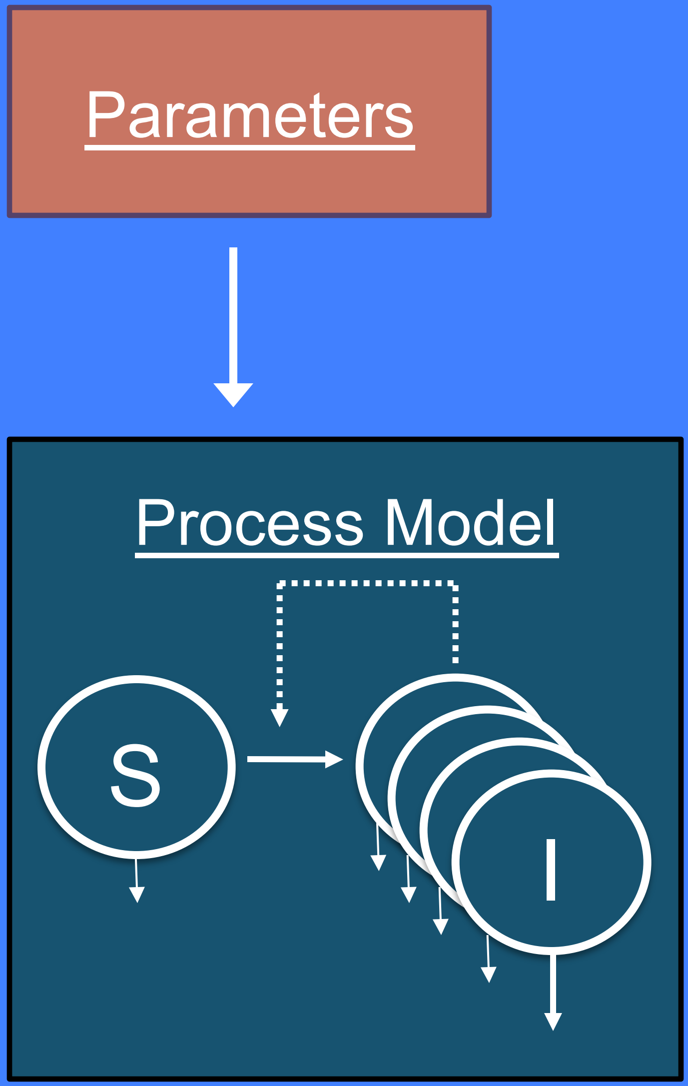


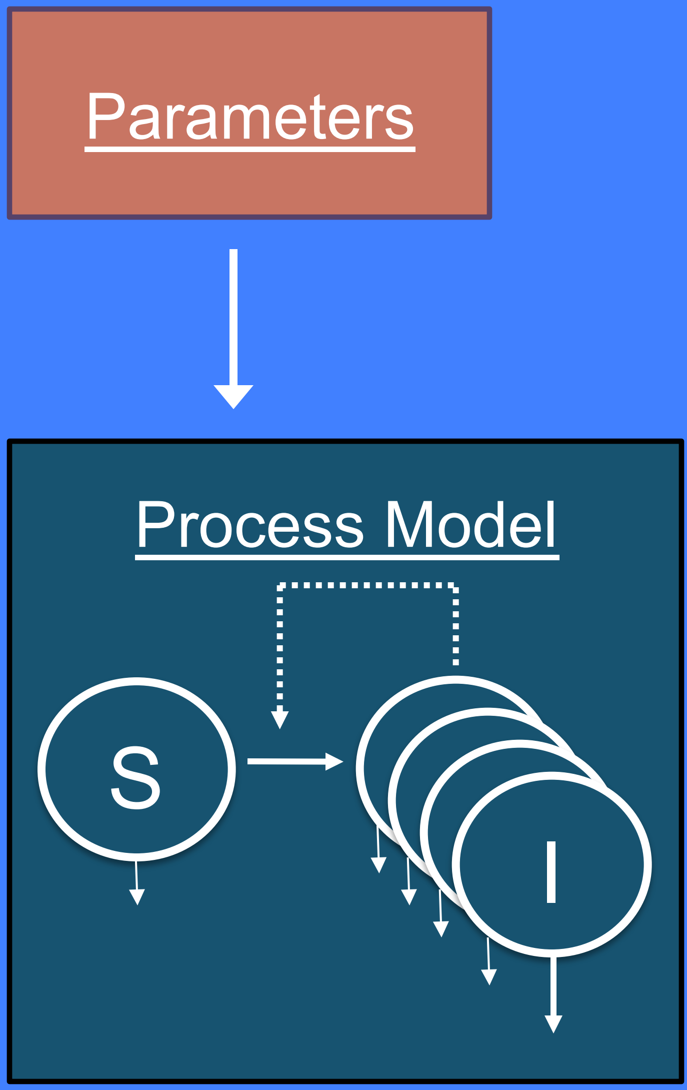
Comparing Confidence Intervals






Process Model
S
I

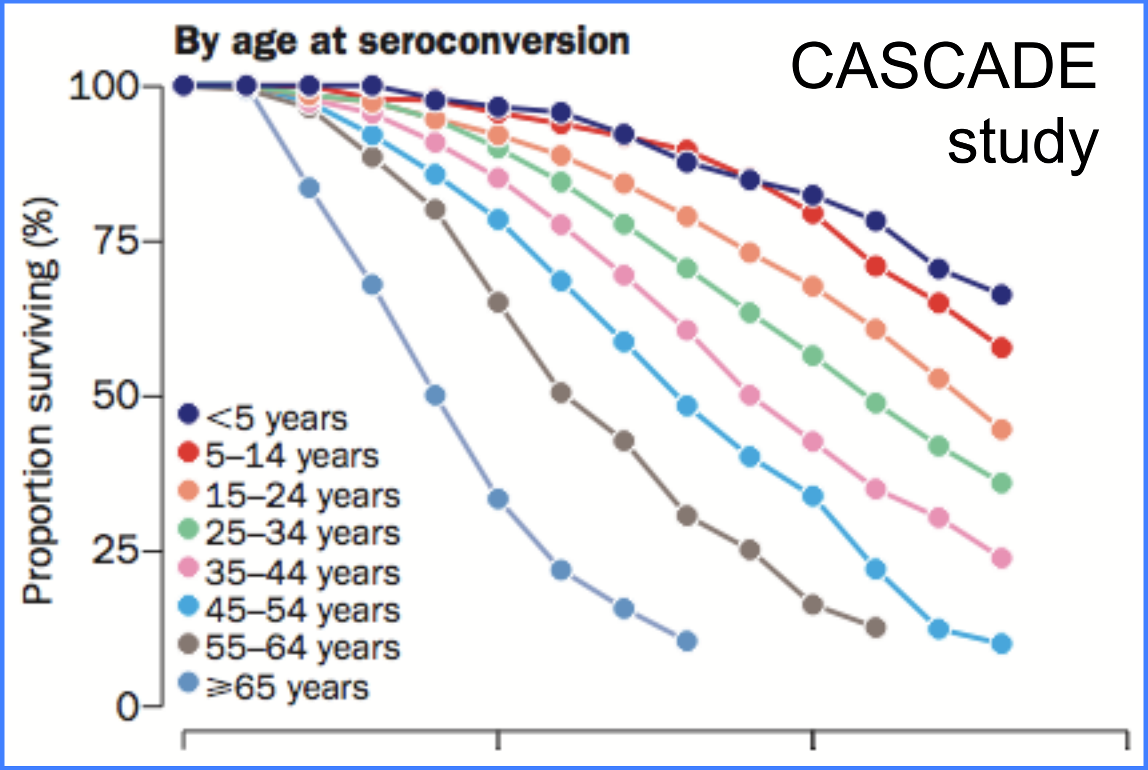

Parameters
Process Model
S
I



Where do parameters come from?

A priori parameterization

- Use external data to determine values for the parameters in your model



A priori parameterization

- Use external data to determine values for the parameters in your model
  - eg, time from seroconversion to death

- Plug estimates into models to determine expected dynamics

A *priori* parameterization

- Long-term time series are not available
- Designing a new study
- Data are limited and your goal is to estimate a particular quantity that has not been directly measured
- Comparing model structures, especially when multiple long-term time series are not available for validation

Fitting models to data

- *A priori* parameterization
  - Use external data to determine values for the parameters in your model
  - Rarely possible for all model parameters

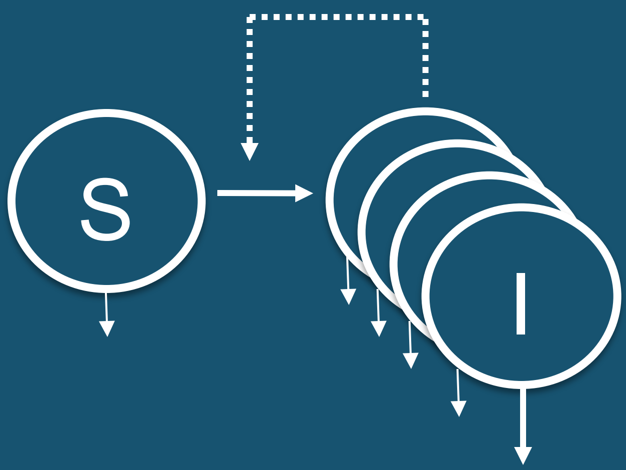
Fitting models to data

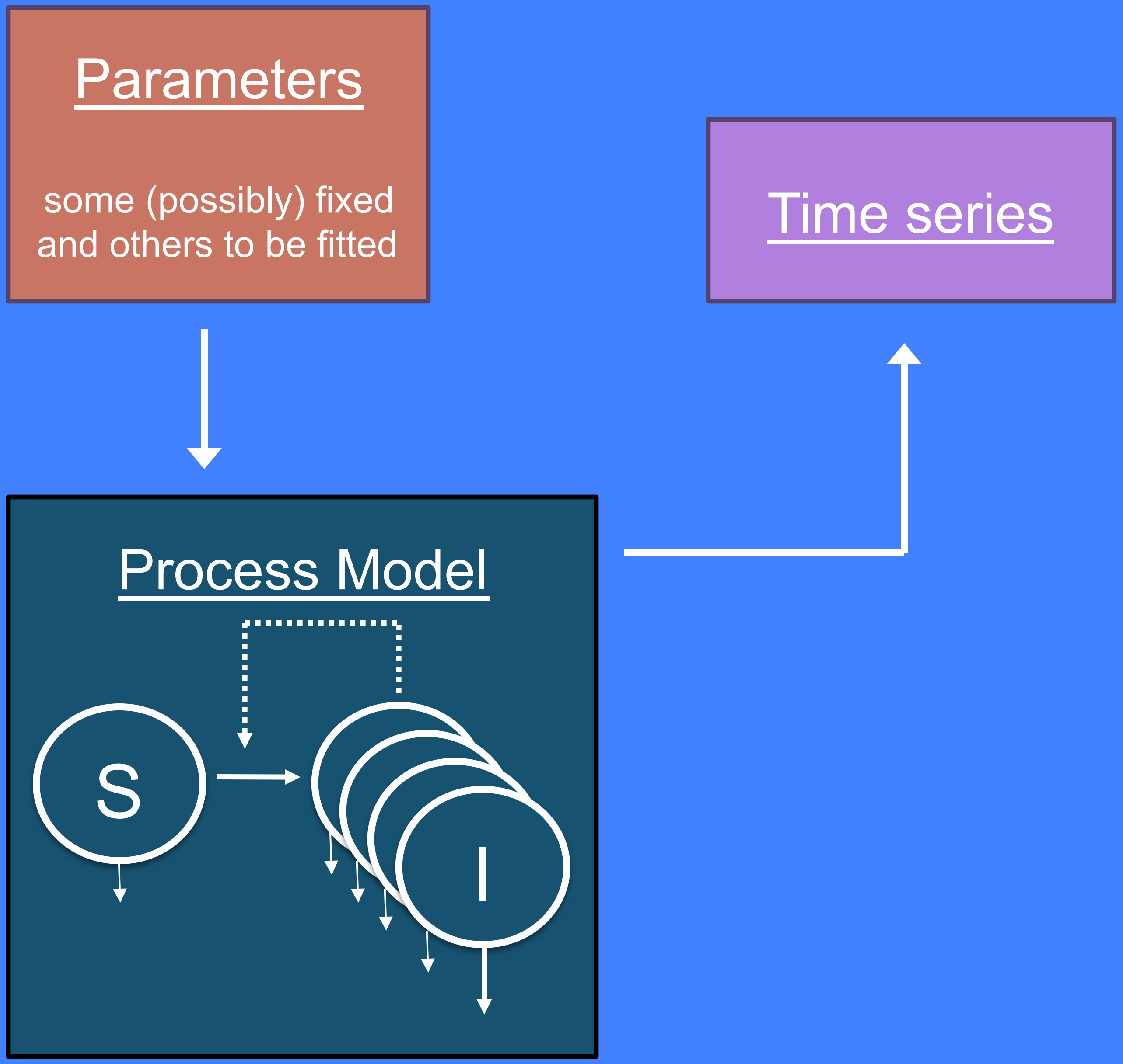
- *A priori* parameterization
  - Use external data to determine values for the parameters in your model
  - Rarely possible for all model parameters
- Trajectory matching
- Feature matching

Parameters

some (possibly) fixed
and others to be fitted

Process Model




Parameters
some (possibly) fixed
and others to be fitted
Time series
Process Model
S
I

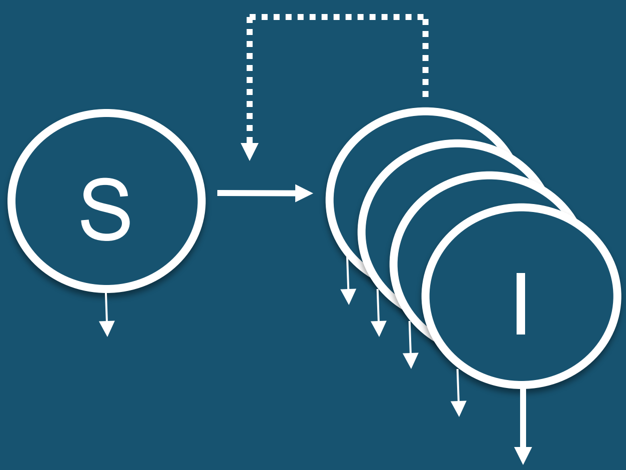
Parameters

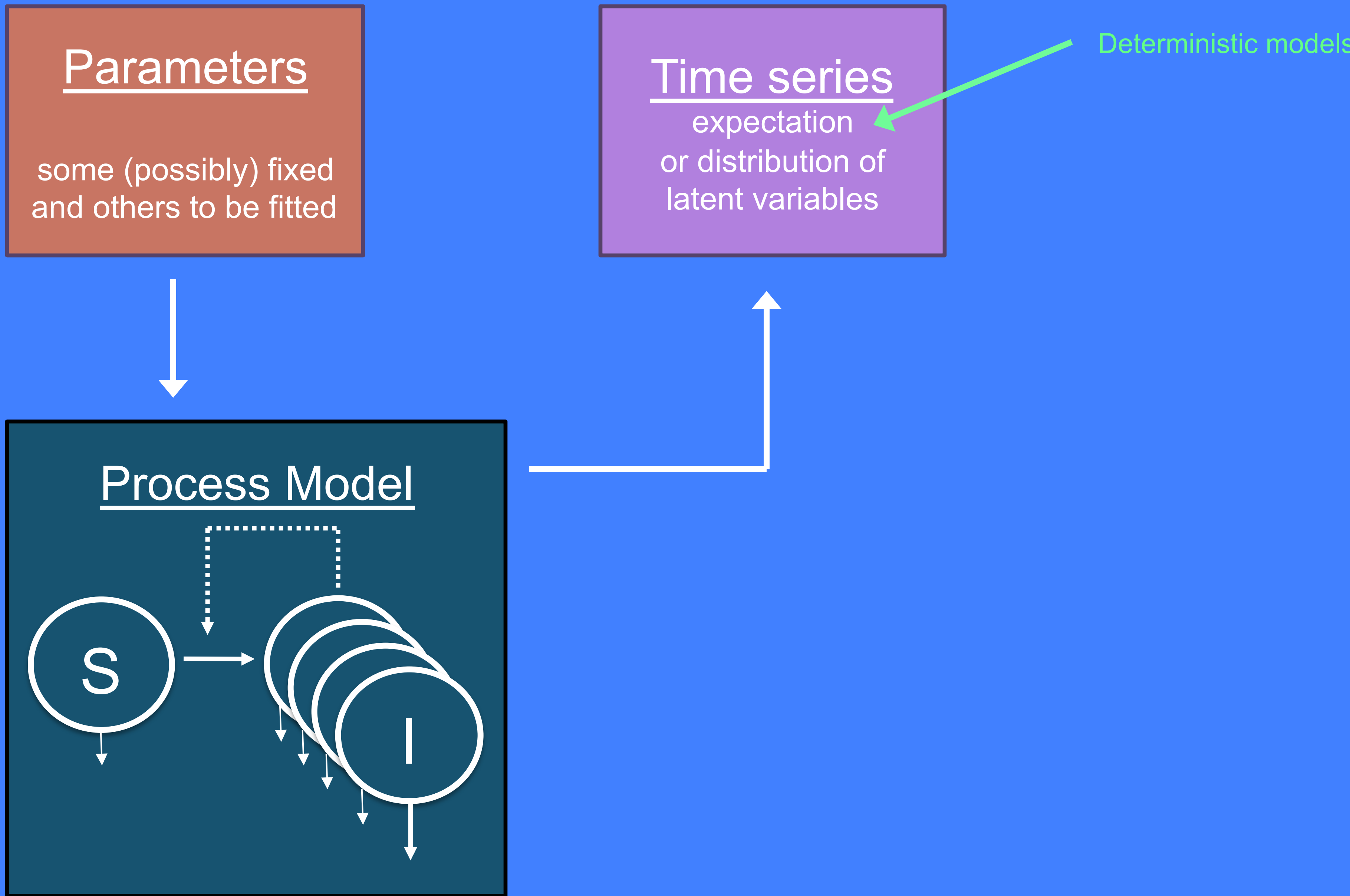
some (possibly) fixed
and others to be fitted

Time series

expectation
or distribution of
latent variables

Process Model




Parameters
some (possibly) fixed
and others to be fitted
Time series
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latent variables
Deterministic models
Process Model
S
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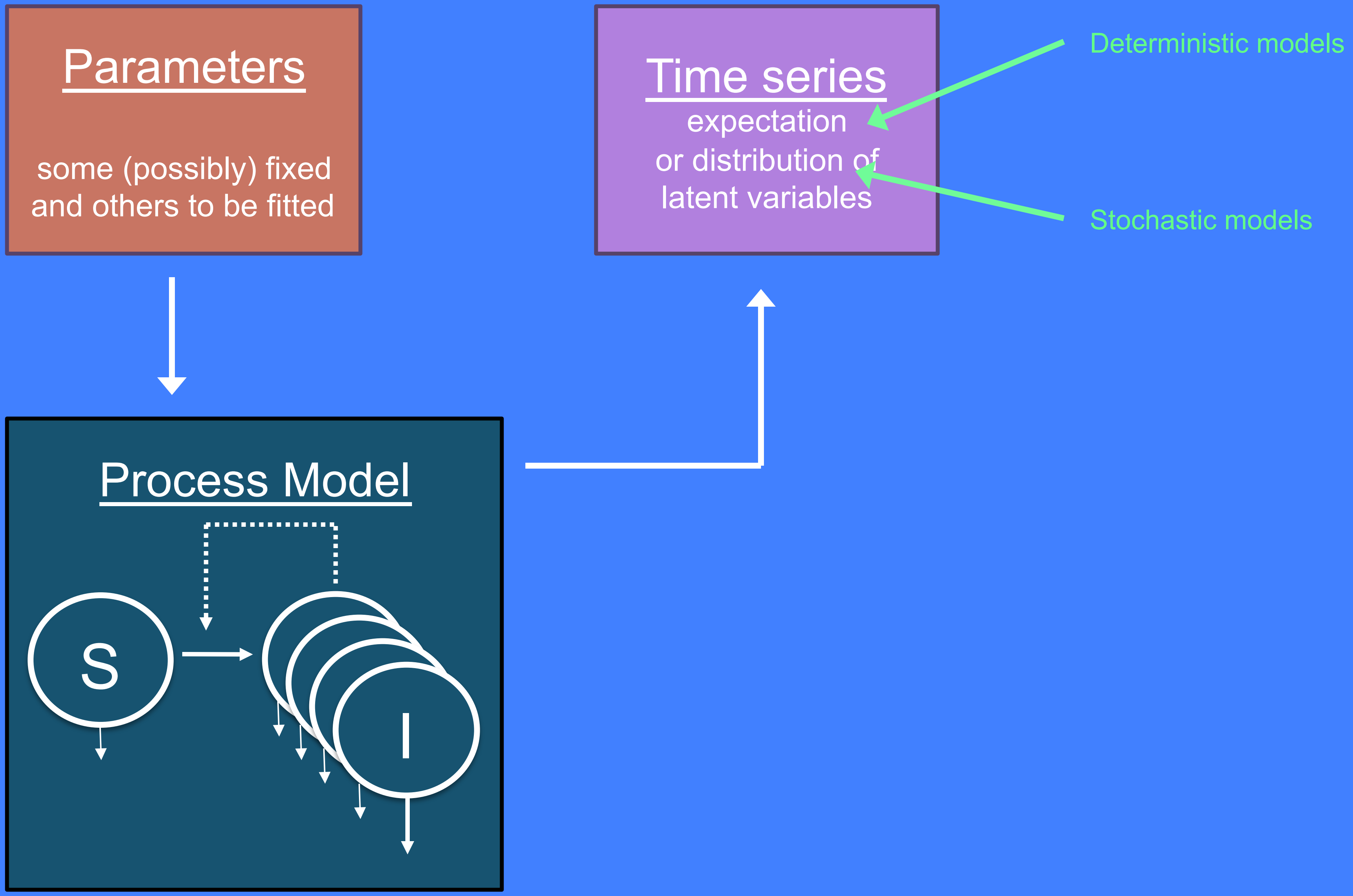
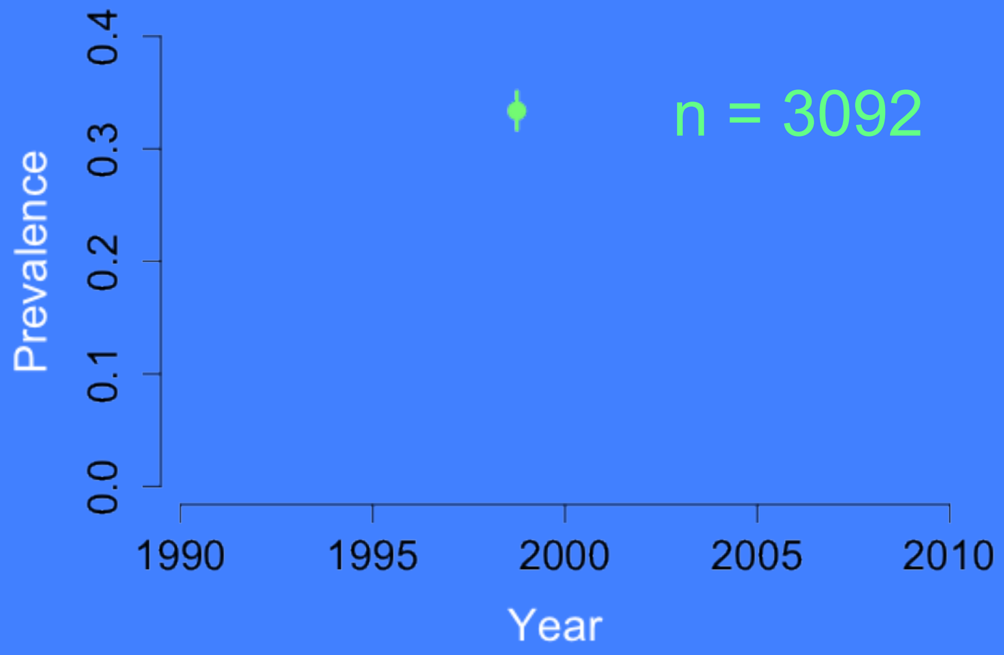
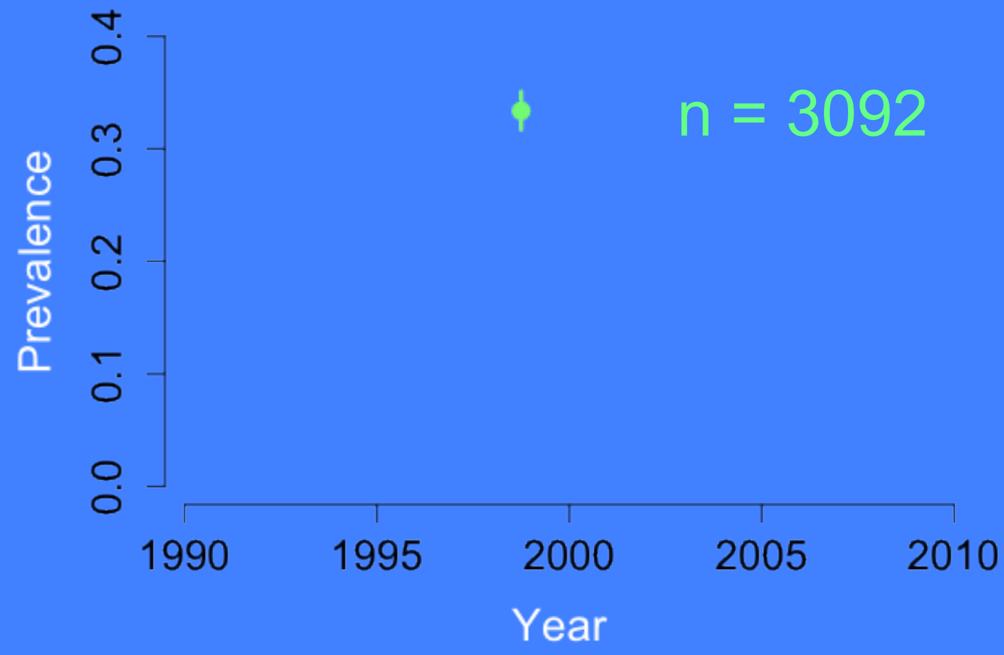

Parameters
some (possibly) fixed
and others to be fitted
Time series
expectation
or distribution of
latent variables
Deterministic models
Stochastic models
Process Model
S
I

Data



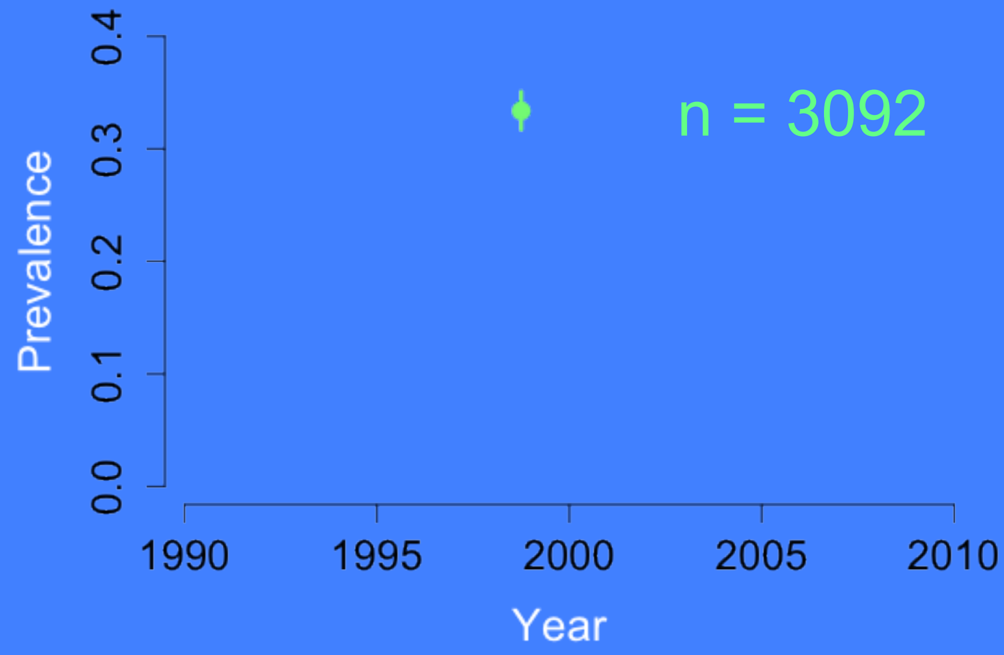
Data



Data

Observation model

PDF: $$f(x \mid p) = \binom{n}{x} p^x (1-p)^{n-x}$$



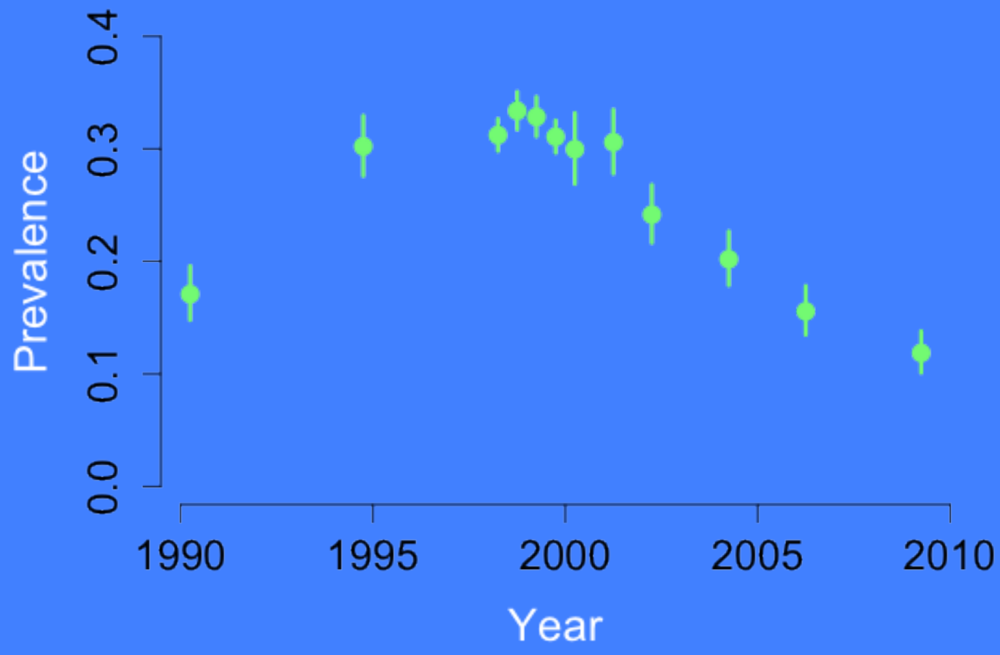
Data

PDF: $$f(x \mid p) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

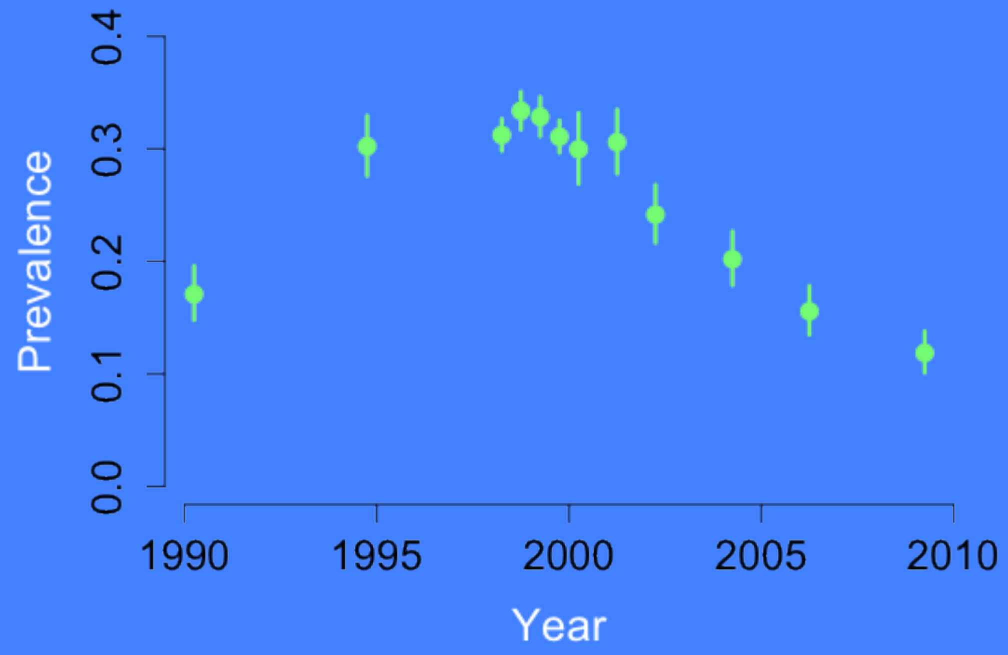
Observation model

LIKELIHOOD: $$L(p \mid x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

Likelihood of prevalence (given data)



Data

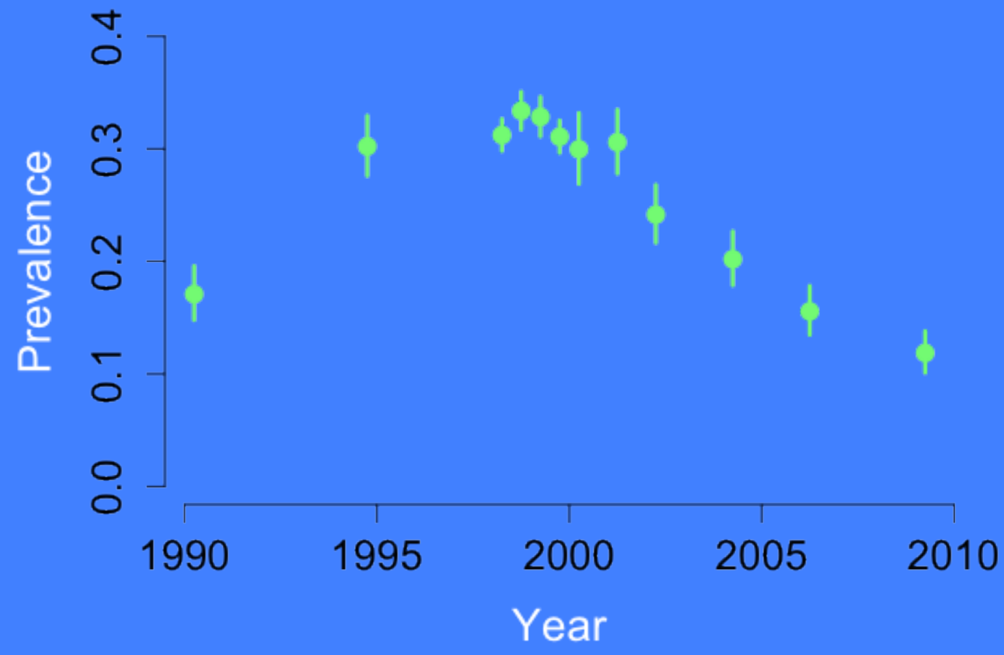


Data

PDF:

$$f(x_t \mid p_t) = \prod_t \binom{n_t}{x_t} p_t^{x_t} (1 - p_t)^{n_t - x_t}$$

Observation model



Data

PDF:

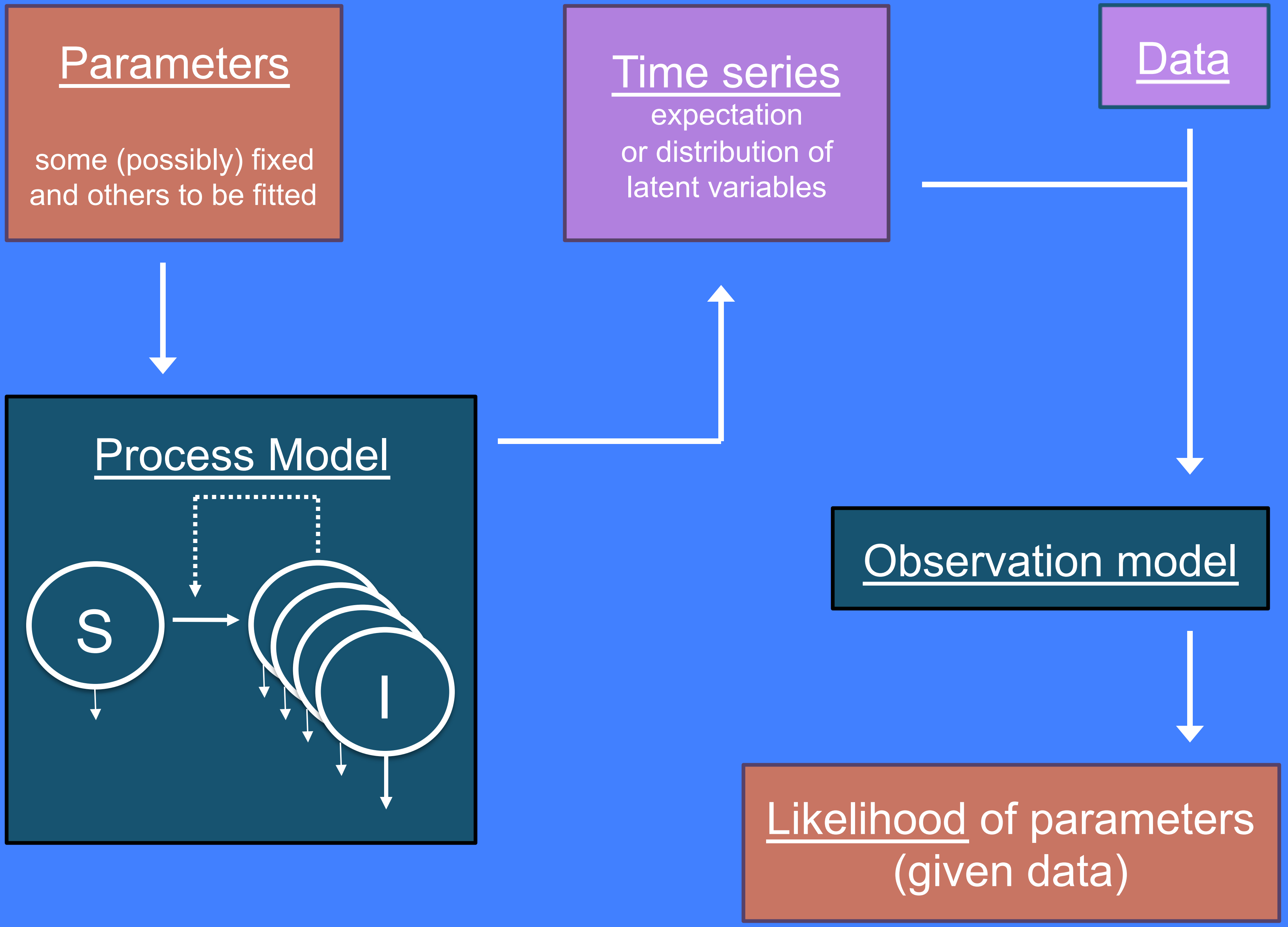
$$f(x_t \mid p_t) = \prod_t \binom{n_t}{x_t} p_t^{x_t} (1 - p_t)^{n_t - x_t}$$

Observation model

LIKELIHOOD:

$$L(p_t \mid x_t) = \prod_t \binom{n_t}{x_t} p_t^{x_t} (1 - p_t)^{n_t - x_t}$$

Likelihood of prevalence trajectory (given data)


Parameters
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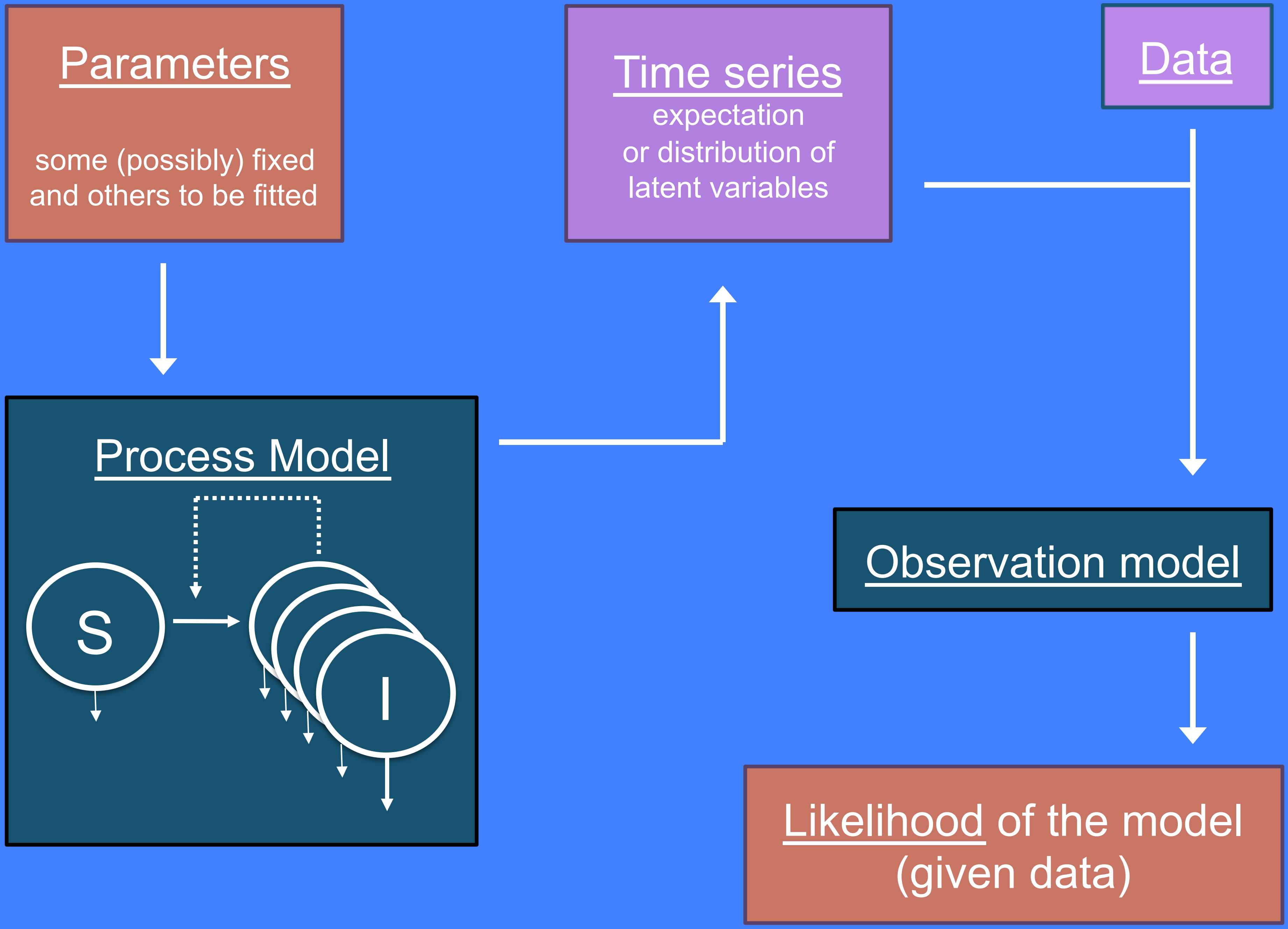
Time series
expectation or distribution of latent variables
Data
Process Model
S
I
Observation model
Likelihood of parameters (given data)


Parameters
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Time series
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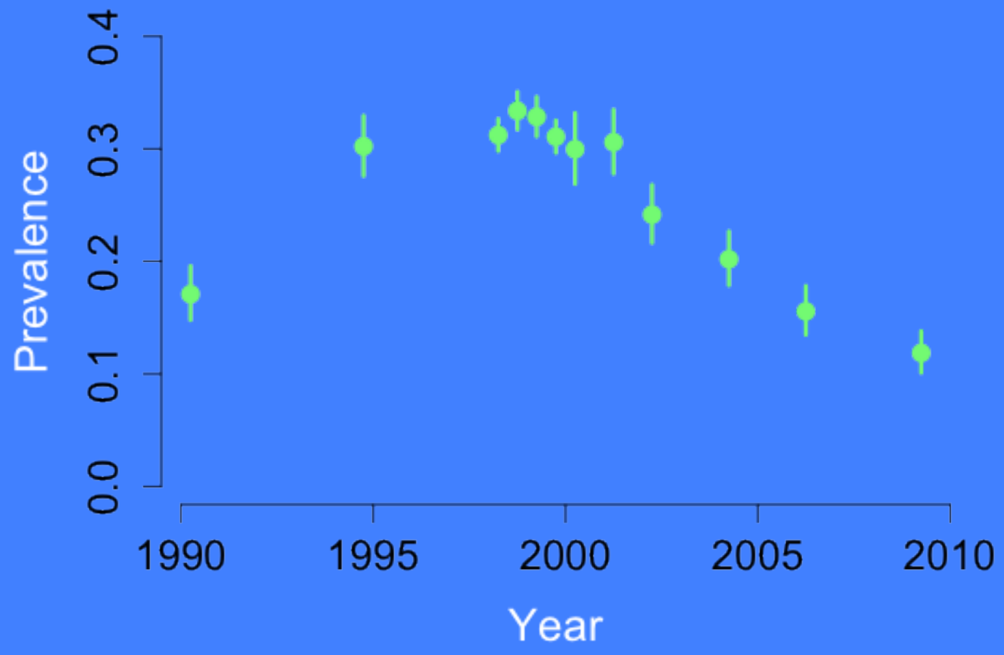
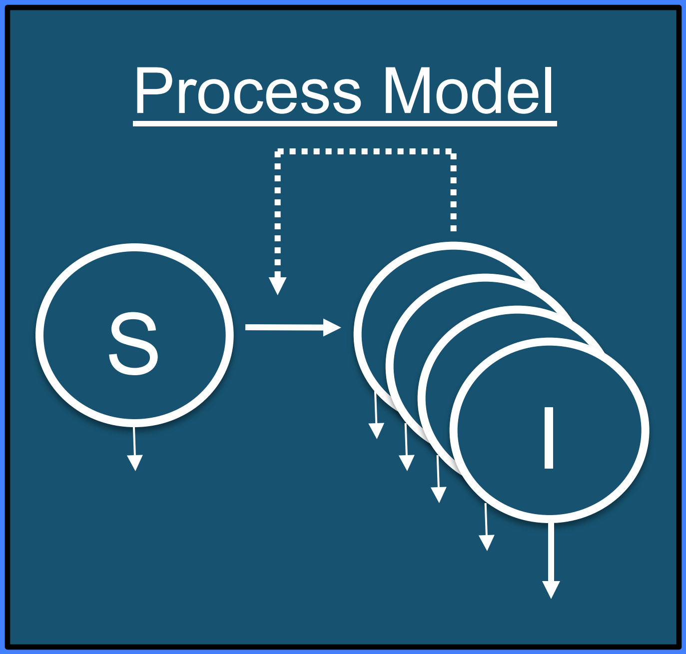
or distribution of
latent variables
Data
Process Model
S
I
Observation model
Likelihood of the model
(given data)





Why do we fit models to data in infectious disease epidemiology?

Koopman’s Inference Robustness Assessment Framework

Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

Koopman's Inference Robustness Assessment Framework

1. Select the policy inference to be pursued

Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

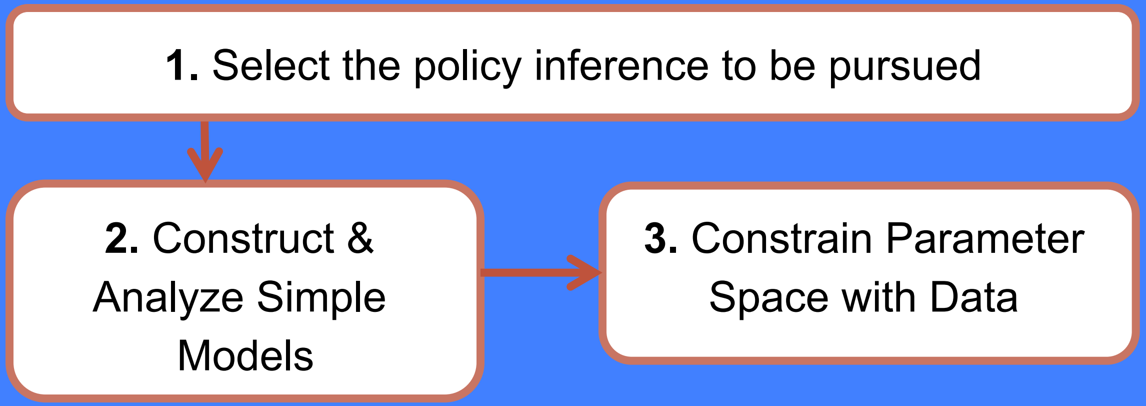
Koopman’ s Inference Robustness Assessment Framework

1. Select the policy inference to be pursued

2. Construct & Analyze Simple Models

Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

Koopman’s Inference Robustness Assessment Framework



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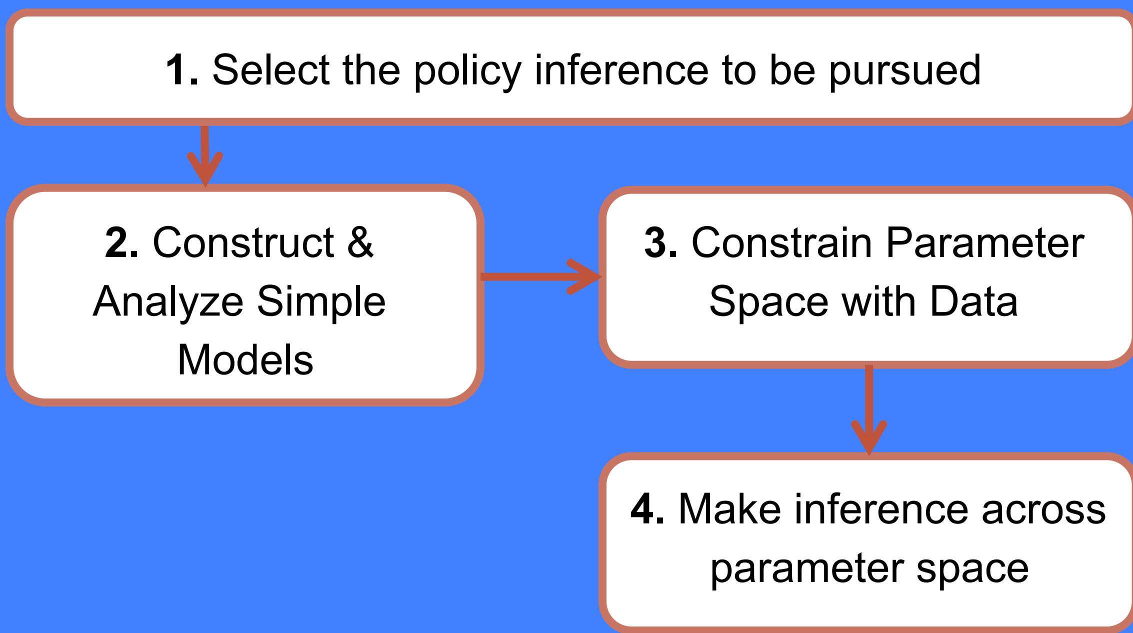

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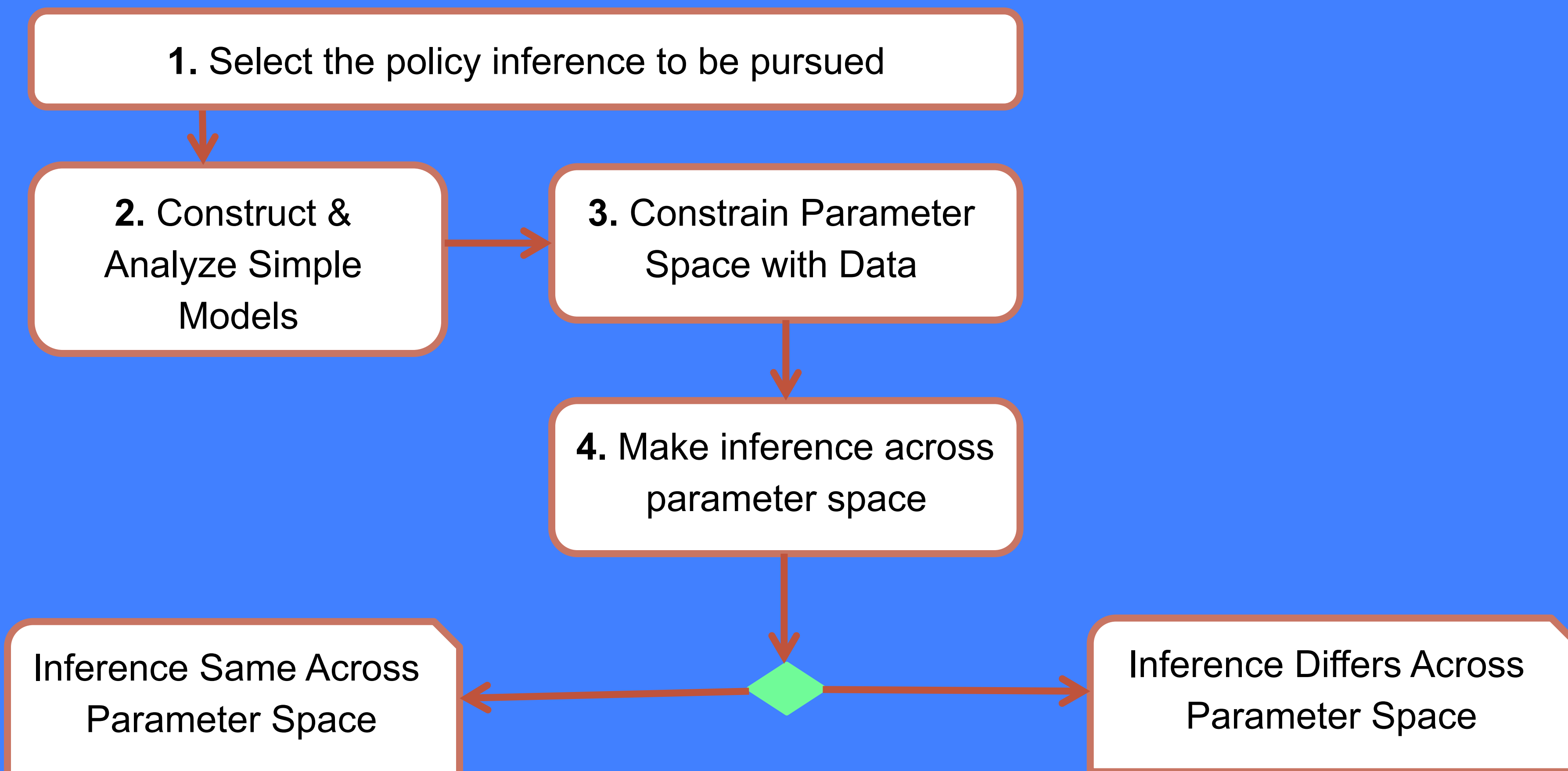

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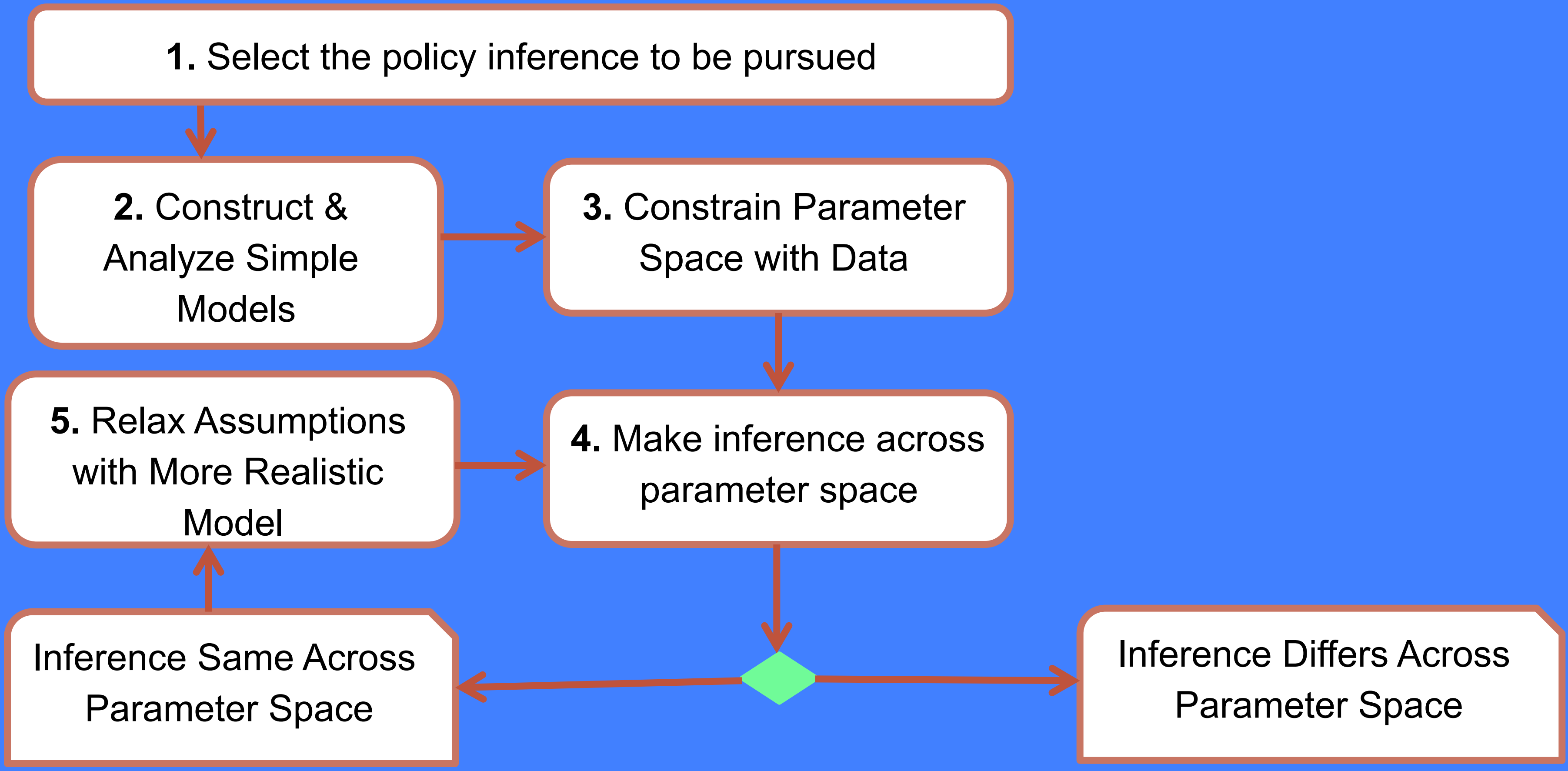

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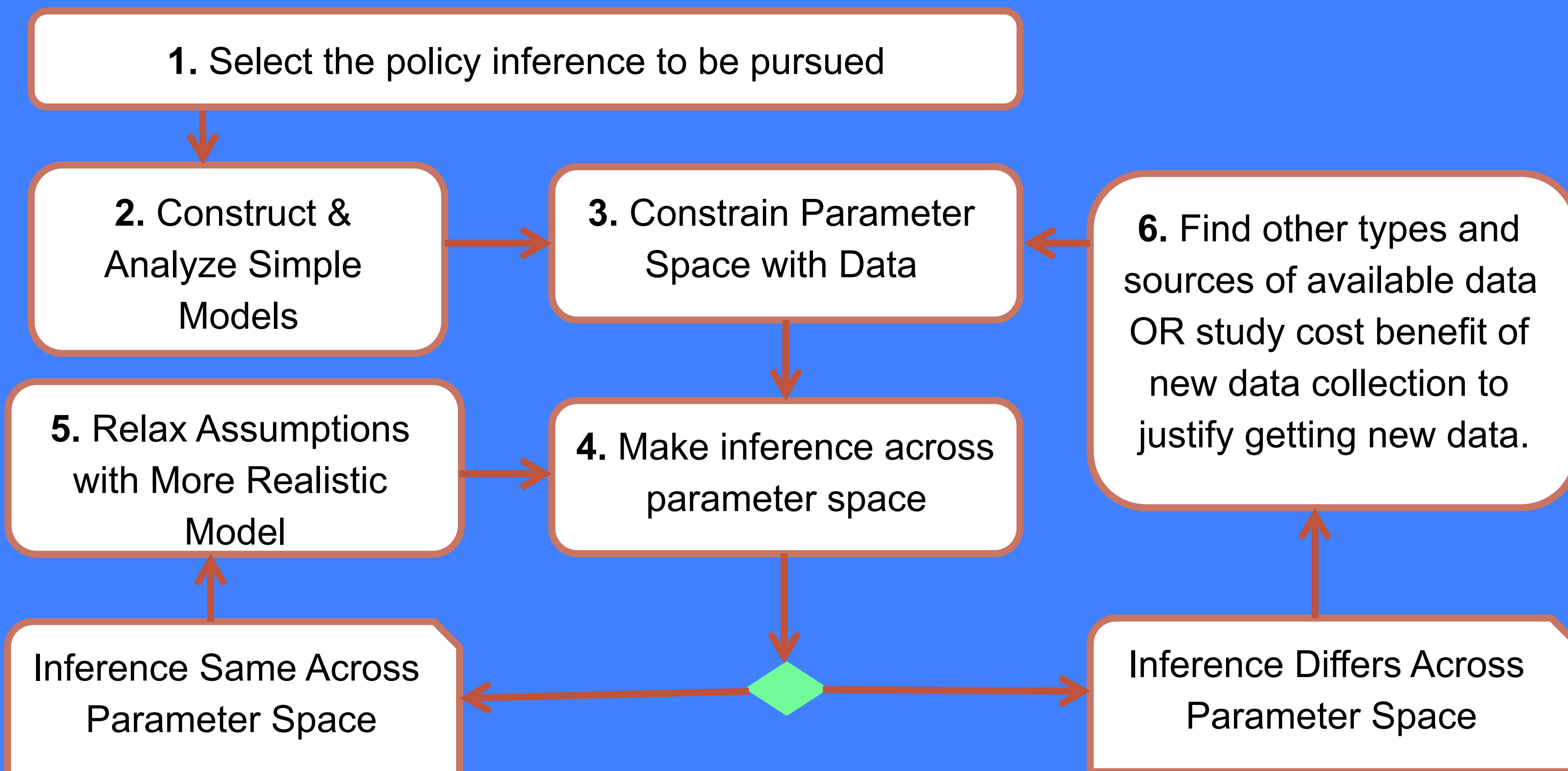
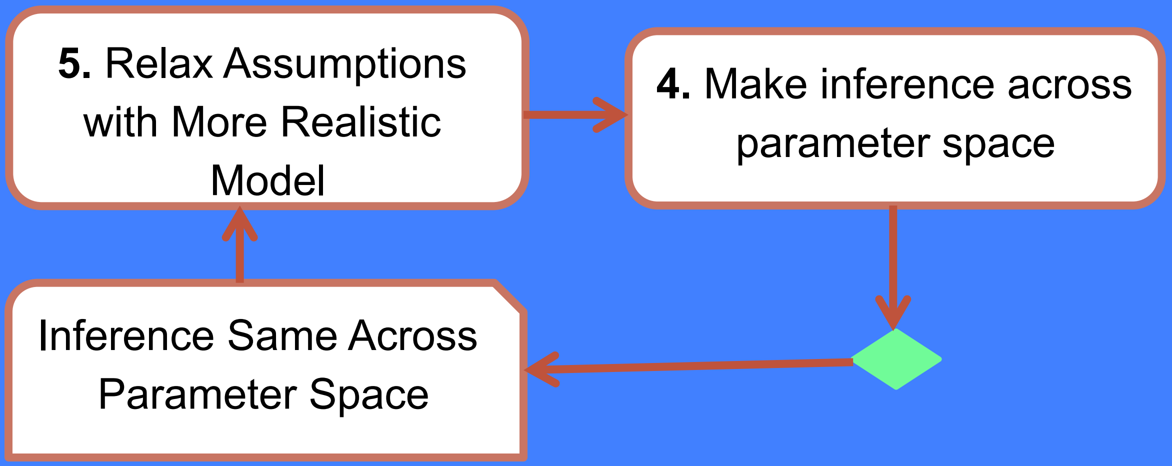


Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

Koopman’s Inference Robustness Assessment Framework

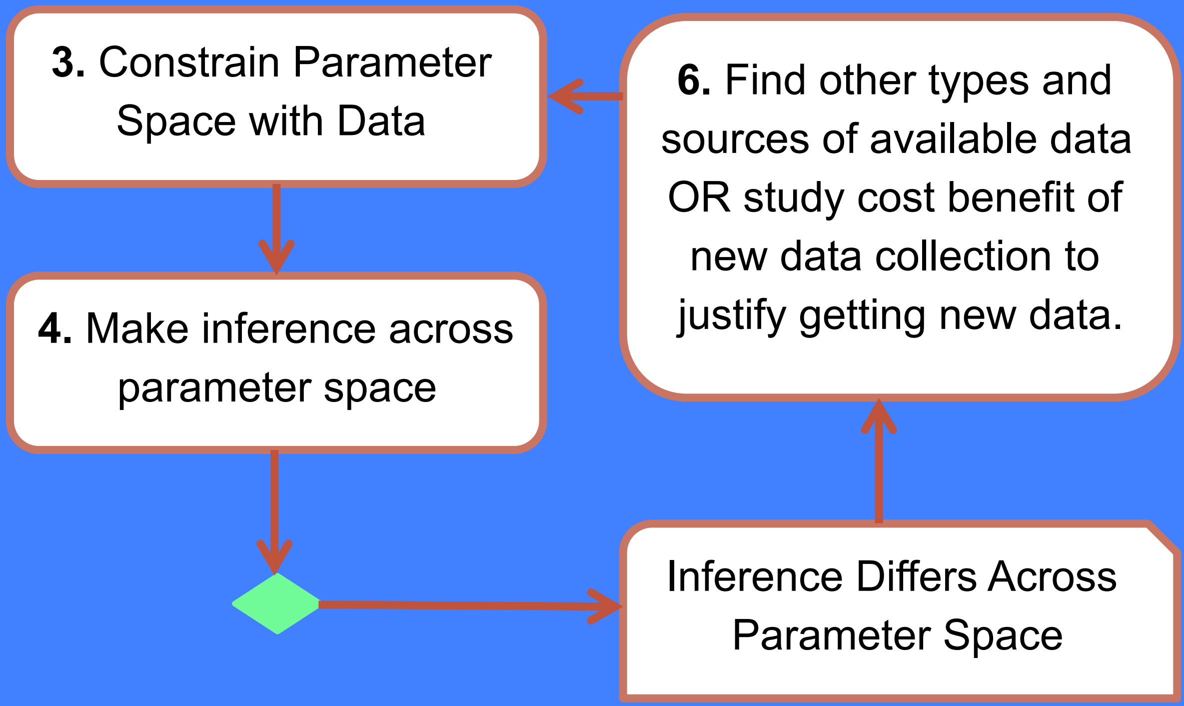
Inference **Robustness** Assessment Loop



Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

Koopman’s Inference Robustness Assessment Framework

Inference **Identifiability** Assessment Loop



Koopman et al. 2014. Transmission modeling to enhance surveillance system function.

Koopman’s Inference Robustness Assessment Framework

- Assess inference robustness to realistic relaxation of simplifying model assumptions

Koopman et al. 2014. Slide courtesy of JS Koopman (with modification).

Koopman’s Inference Robustness Assessment Framework

- Assess inference robustness to realistic relaxation of simplifying model assumptions
- Pursue complexity that matters by keeping models as simple as possible but not ***so*** simple that they lead to an incorrect inference

Koopman et al. 2014. Slide courtesy of JS Koopman (with modification).

Koopman's Inference Robustness Assessment Framework

- Assess inference robustness to realistic relaxation of simplifying model assumptions
- Pursue complexity that matters by keeping models as simple as possible but not ***so*** simple that they lead to an incorrect inference

Validate the inference!

Koopman et al. 2014. Slide courtesy of JS Koopman (with modification).

Koopman's Inference Robustness Assessment Framework

- Assess inference robustness to realistic relaxation of simplifying model assumptions
- Pursue complexity that matters by keeping models as simple as possible but not ***so*** simple that they lead to an incorrect inference

Validate the inference!

not the model or method you're working with

Koopman et al. 2014. Slide courtesy of JS Koopman (with modification).

International Clinics on Infectious Disease Dynamics and Data: The ICI3D Program

SACEMA
DST/NRF Centre of Excellence in Epidemiological Modelling and Analysis

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