Reed-Frost Models

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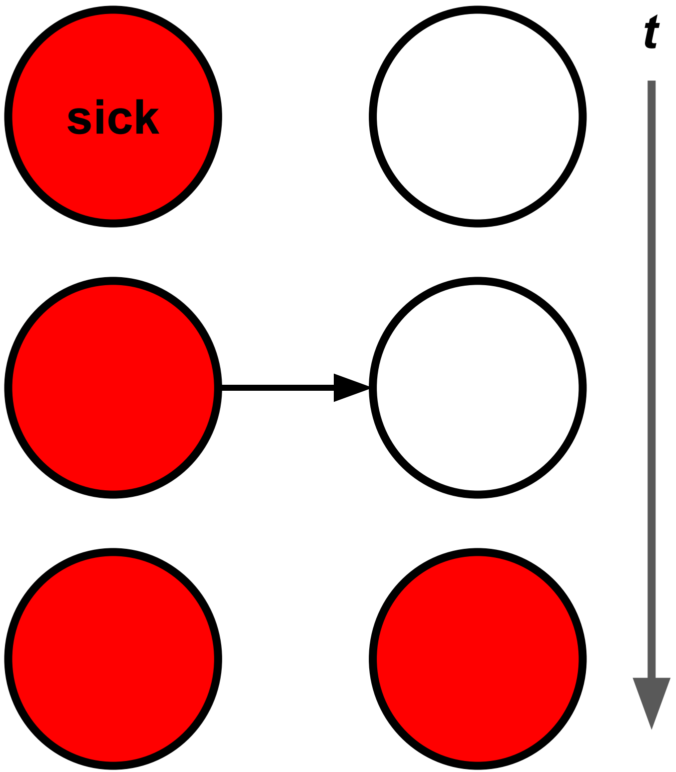
White Oak

What is the simplest, formal model of transmission you can propose?

A Minimal Model

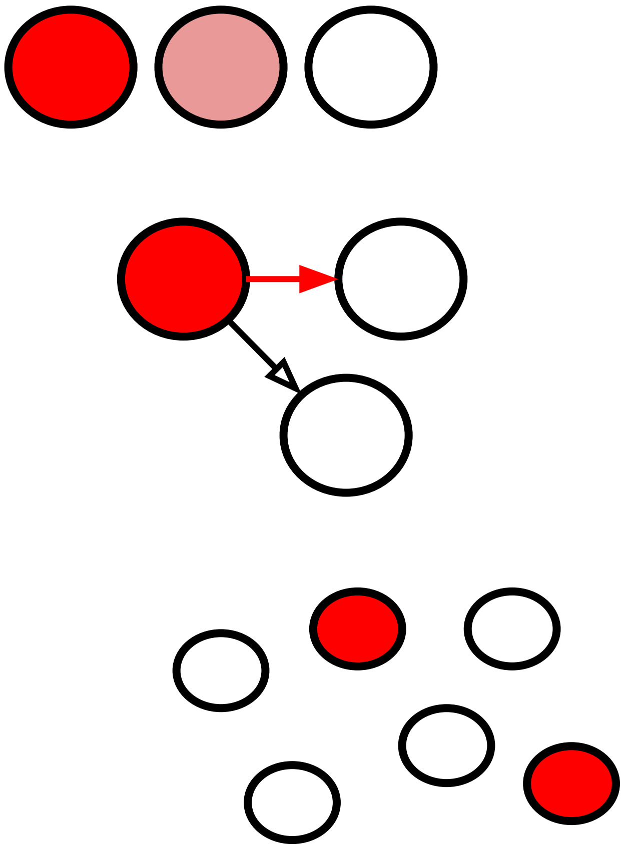
- Hosts are either sick or not sick
- Time passes in fixed steps
- All hosts interact each step
- If a sick host interacts with a not sick host, the not sick host is sick on the next time step
- Simplest instance, 2 hosts. Starts: neither sick, both sick, one sick + one not.



Using my model, what are single changes that make the model more useful?

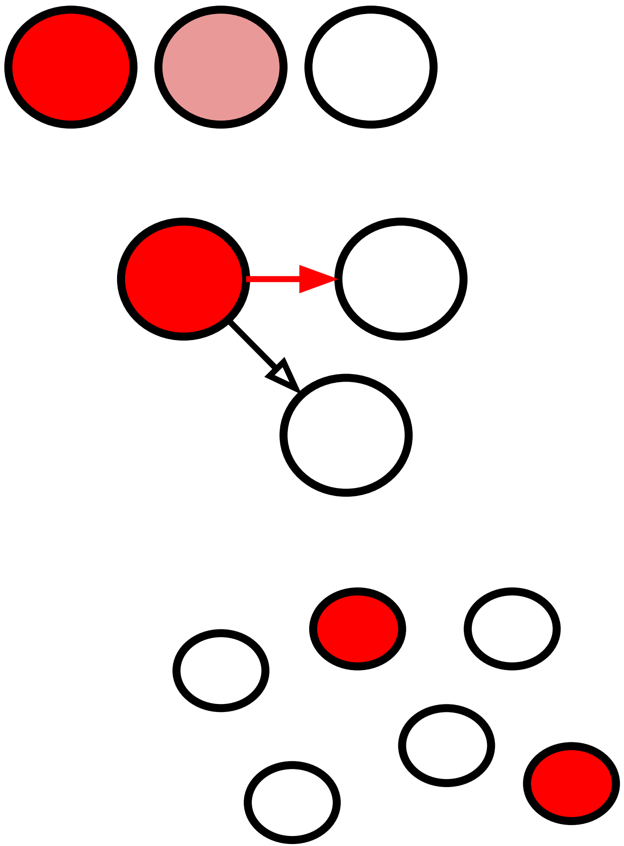
Less Minimal Models

- Hosts are either sick, not sick, or some other condition
- Time passes in different durations
- Some hosts interact each step
- Interaction may or may not lead to infection
- More than two hosts
- Becoming sick takes multiple steps
- Et cetera...



Less Minimal Models

- **Hosts are either sick, not sick, or *removed***
- Time passes in different durations
- Some hosts interact each step
- **Interaction may or may not lead to infection**
- **More than two hosts**
- Becoming sick takes multiple steps



Reed-Frost Model

- There is a population, *N*, of hosts
- Time proceeds in fixed steps
- Hosts are either a case, susceptible, or removed
- Each time step, all hosts interact
- When a case and susceptible host interact during a time step, there is a probability *p* that the susceptible host will become a case on the next time step
- All cases at some time step become immune on the next time step

MID-DAY BREAK

Reed-Frost Model Math

- All interactions instantaneous, independent
- Host counts at time steps: C_t, S_t, R_t
- $R_{t+1} = C_t + R_t$ -- i.e., all infectious become immune
- For a particular susceptible,
  $P(\text{infected @ } t+1 \mid C_t) = 1 - (1-p)^C = 1\text{-}q^C$
- Therefore,
  $P(S_{t+1} = S_t - x,\ C_{t+l} = x \mid S_t,\ C_t) = \binom{S_t}{x} (1\text{-}q^C)^x (q^C)^{S\text{-}x}$

- Not easy to turn into final size calculations

EXPLORING REED-FROST WITH R